DATE KEY!

NAME

4.2 Concepts Worksheet

Implicit Differentiation

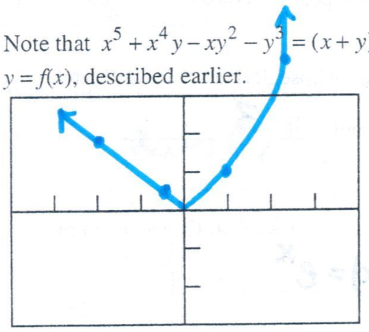
Let $y = f(x)$ be the continuous function satisfying the equation $x^5 + x^4y - xy^2 - y^3 = 0$ and containing the points $\left(-\frac{1}{2}, \frac{1}{2}\right)$, $(-2, 2)$, and $(2, 4)$.

1. Find an expression for $\frac{dy}{dx}$ in terms of x and y. $\frac{dy}{dx} = \frac{y^2 - 5x^4 - 4x^3y}{x^4 - 2xy - 3y^2}$

2. Find $\frac{dy}{dx}$ at each of the following points.

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ -1 (b) $(-2, 2)$ -1 (c) $(2, 4)$ 4

3. Note that $x^5 + x^4y - xy^2 - y^3 = (x+y)(x^2+y)(x^2-y)$. Use this factorization to graph the function $y = f(x)$, described earlier.



$x+y=0$ line
$y=-x$
$y'=-1$

$\cancel{x^2+y=0}$
$\cancel{y=-x^2}$
$\cancel{y'=-2x}$
the given pts. do not fit this equation.

$x^2-y=0$
$y=x^2$ parabola
$y'=2x$

4. Can the expression you wrote in question 1 above be used to find $\frac{dy}{dx}$ at $(0, 0)$? Explain.

No, the $\frac{dy}{dx}$ expression is not defined at $(0,0)$.

5. Evaluate $\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$. 0

6. Evaluate $\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$. -1

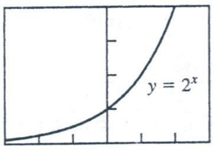
7. Can you evaluate $f'(0)$? No. Why or why not? The right & left-hand limits at $x=0$ are not equal.

4.2 Concepts Worksheet *Continued* NAME________________________

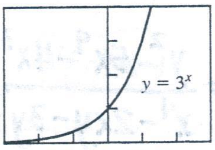
Concept Connectors

We have yet to discuss the calculus of exponential and logarithmic functions. However, a sense of their differentiation could be developed geometrically:

Note the following graphs of exponential functions.



[−3, 3] by [0, 4]



[−3, 3] by [0, 4]

Imagine a sketch of the "derivative" graphs on the curves above. Since precise range values of the derivative are still open to interpretation using this technique, it may appear that the derivative graphs for 2^x and 3^x are much like the curves themselves. In fact, the derivative of 2^x is the product of 2^x and some positive constant $x = 0.693$, which is less than 1, while the derivative 3^x is the product of 3^x and some positive constant $d = 1.09$, which is greater than 1.

8. Hence, one might conclude that there exists an exponential function whose derivative is itself, and that the base of this exponential function lies between ____2____ and ____3____.

9. Surely, a beautiful curve in calculus would be one whose derivative curve is itself. Cite an example, previously studied, of a function whose derivative is itself. $f(x) = e^x$

DATE____________________

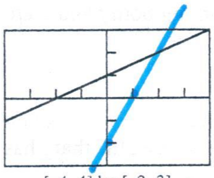
4.3 Concepts Worksheet

NAME____________________

Inverse Functions

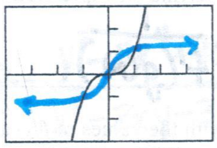
Given the graph $f(x)$, roughly sketch $f^{-1}(x)$ on the same grid.

1.



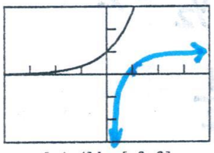
[−4, 4] by [−3, 3]

2.



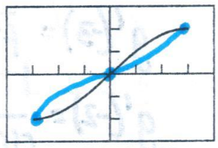
[−4, 4] by [−3, 3]

3.



[−4, 4] by [−3, 3]

4.

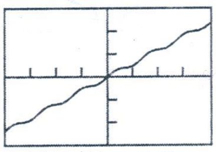


[−4, 4] by [−3, 3]

5. Which of the following have inverse functions?

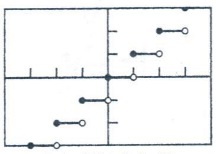
a, c, e, f
____

a.



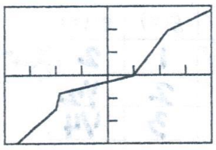
[−4, 4] by [−3, 3]

b.



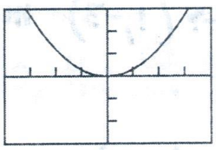
[−4, 4] by [−3, 3]

c.



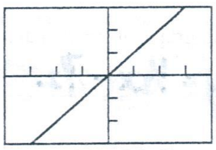
[−4, 4] by [−3, 3]

d.



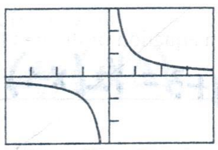
[−4, 4] by [−3, 3]

e.



[−4, 4] by [−3, 3]

f.



[−4, 4] by [−3, 3]

6. Which of the functions in Problem 5 appear to have an inverse function whose graph is the same as the original graph?

e, f
____

____

4.3 Concepts Worksheet *Continued* NAME_______________

Concept Connectors

7. If $f(g(x)) = g(f(x)) = x$, what is the relationship between functions f and g?

f(x) & g(x) are inverses.

8. Use implicit differentiation to find an expression for $g'(x)$ using $f(g(x)) = x$, assuming both f and g are differentiable.

$g'(x) = \frac{1}{f'(g(x))}$

Let f be a differentiable function with the values of $f(x)$ and $f'(x)$ given in the table below. Assume that f has a differentiable inverse function, $g(x) = f^{-1}(x)$.

x	$f(x)$	$f'(x)$
1	−3	$\frac{1}{2}$
2	−2	2
3	1	4

$g'(-3) = \frac{1}{f'(g(-3))} = \frac{1}{f'(1)} = \frac{1}{1/2} = 2$

$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(2)} = \frac{1}{2}$

$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(3)} = \frac{1}{4}$

9. Complete the table to give as much information as possible about the inverse function.

x	$g(x)$	$g'(x)$
−3	1	2
−2	2	1/2
1	3	1/4

10. Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 1$. → $(1,-3)$ $m = 1/2$

$y+3 = 1/2(x-1)$ OR $y = 1/2x - 7/2$

11. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 1$. → $(1,3)$ $m = 1/4$

$y-3 = 1/4(x-1)$ OR $y = 1/4x + 11/4$

12. Find an equation of the line $\boxed{\text{normal}}$ to the graph of $y = g(x)$ at $x = -2$. → $(-2,2)$ $m = 1/2$

$\perp m = -2$

$y-2 = -2(x+2)$ OR $y = -2x-2$