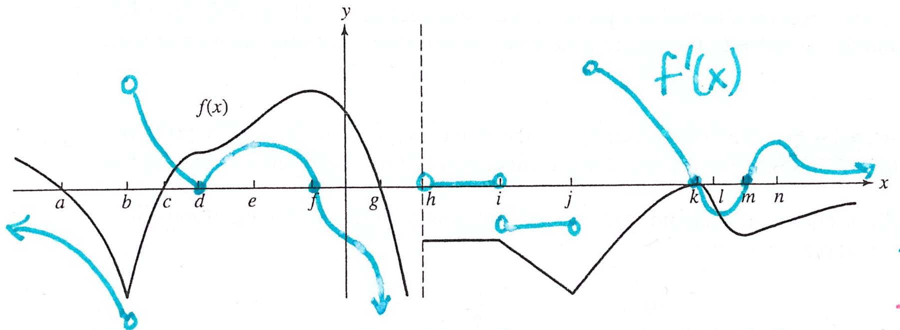
DATE KEY

## 5.1 Concepts Worksheet

NAME

### An Unusual Function




* Book Answers
* Ones we are questioning

1. The function $f$ drawn above would be difficult to describe algebraically; nevertheless, it has interesting geometric features for which calculus provides descriptions. Using the textbook definitions and some freedom of artistic judgment, name the value(s) of $x$ for:

   (a) zeros of $f(x)$ a, c, g, k

   (b) points of discontinuity of $f$ h

   (c) points where $f'(x)$ does not exist b, h, j, i

   (d) points where $f'(x) = 0$ f, m, k, d, $h<x<i$

   (e) critical points b, d, f, $h \le x \le i$, j, k, m

   (f) critical points that are not stationary points b, h, j, i

   (g) intervals over which $f$ increases $(b,f) \cup (j,k) \cup (m,\infty)$

   (h) intervals over which $f$ decreases $(-\infty,b) \cup (f,h) \cup (i,j) \cup (k,m)$

   (i) relative maxima ? d, f, k, $h \le x \le i$

   (j) absolute maxima f (None)

   (k) relative minima m, b, j, $h \le x \le i$

   (l) absolute minima None

2. (a) Find the equation of any horizontal asymptotes.

   None $y=0$

   (b) Find the equation of any vertical asymptote(s).

   $x=h$

3. Find the $x$-coordinate of each point of discontinuity of $f'$. b, h, i, j

4. Find the $x$-coordinate of each critical point of $f'$. b, d, e, $h \le x \le i$, j, l, n

5. Sketch $f'$ on the same graph as $f$. (You will need to approximate the range extent of $f'(x)$ as you graph.)

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## 5.2 Concepts Worksheet

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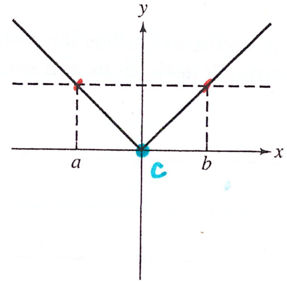
**Theorems of Calculus**

Rolle's Theorem states: If a function is continuous at every point on a closed interval $[a, b]$ and differentiable on every point of its interior $(a, b)$ and $f(a) = f(b) = 0$, then there is at least one number $c$ between $a$ and $b$ at which $f'(c) = 0$.

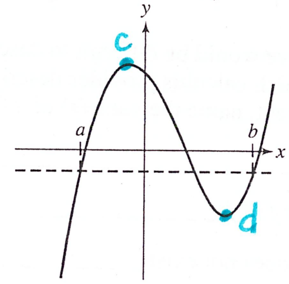
A variation of Rolle's Theorem includes broader conditions: If a function $f(x)$ is continuous at every point in a closed interval $[a, b]$ and $f(a) = f(b)$, then there exists at least one critical point of $f(x)$ between $x = a$ and $x = b$.

1. Using this variation of Rolle's Theorem, find and mark the critical points on the following graphs, if applicable. If not applicable, explain why not.

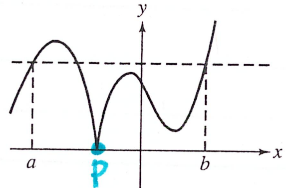
(a)




(b)

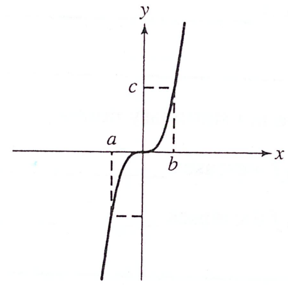



(c)




Not differentiable on $(a,b)$

(d)

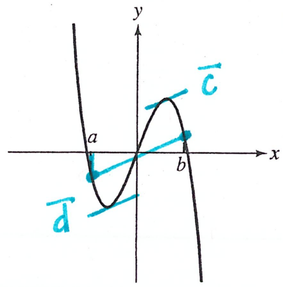



Not applicable because $f(a) \neq f(b)$

## 5.2 Concepts Worksheet Continued NAME________________

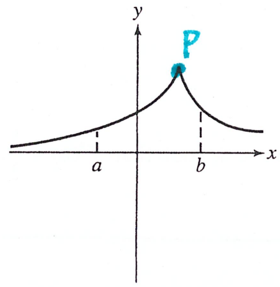
**2.** Given the functions below as drawn over the interval $[a, b]$ are the conditions of the Mean Value Theorem met? (If not, why not?) If conditions are met, locate the value(s) of $c$ that satisfy the equation $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. Draw the parallel tangent lines and secant line implied by the Mean Value Theorem.

**(a)**



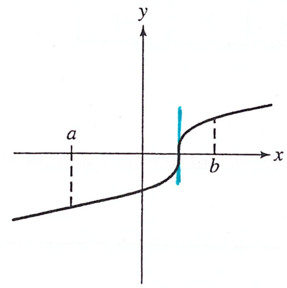

Yes, conditions are met. c̄ and d̄ are // to the secant line $(\overline{ab})$

**(b)**



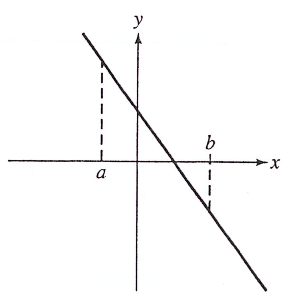

No, not differentiable on (a,b) @ pt p.

**(c)**



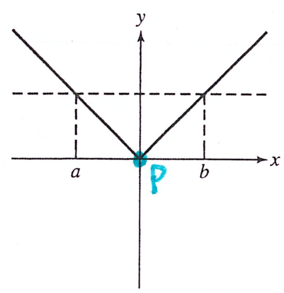

No, it has a vertical tangent in the interval

**(d)**




Yes, all pts on (a,b) will be // to secant line $(\overline{ab})$

**(e)**




No, not differentiable @ pt. p.

Concept Connectors

3. Suppose $f(x)$ is a function with continuous first and second derivatives on the closed interval [0, 4] and whose values for $f$ and $f'$ at $x = 0$ and $x = 4$ are given below:

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 0 | 3 | 3 |
| 4 | 5 | $-1$ |

(a) Prove there exists a value of $c$, $0 < c < 4$, such that $f'(c) = \frac{1}{2}$.

Use MVT: $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\frac{1}{2} = \dfrac{5-3}{4-0}$

$\frac{1}{2} = 2/4$ ✓

(b) Prove there exists a value of $d$, $0 < d < 4$, such that $f''(d) = -1$.

Use MVT: $f''(d) = \dfrac{f'(b)-f'(a)}{b-a}$

$-1 = \dfrac{-1-3}{4-0}$

$-1 = \dfrac{-4}{4}$ ✓

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## 5.3 Concepts Worksheet

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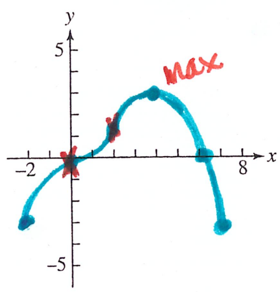
### Graph Sketching Using Derivatives

1. Sketch a graph of a differentiable function $f(x)$ over the closed interval $[-2, 7]$, where $f(-2) = f(7) = -3$ and $f(4) = 3$. The roots of $f(x) = 0$ occur at $x = 0$ and $x = 6$, and $f(x)$ has properties indicated in the table below:

| | | pt of infl. | | pt of infl. | | max | |
|---|---|---|---|---|---|---|---|
| $x$ | $-2 < x < 0$ | $x = 0$ | $0 < x < 2$ | $x = 2$ | $2 < x < 4$ | $x = 4$ | $4 < x < 7$ |
| $f'(x)$ | positive inc | 0 | positive inc | 1 | positive inc | 0 | negative dec |
| $f''(x)$ | negative | 0 | positive | 0 | negative | 0 | negative |
| | ccd | | cc up | | ccd | | ccd |




roots @ $x=0$ & $x=6$

$\downarrow$

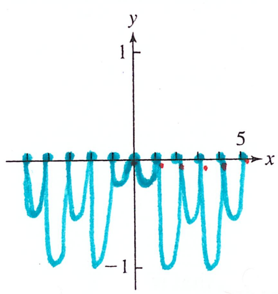
$(0,0)$ $(6,0)$

pts: $(-2,-3)$ & $(7,-3)$ & $(4,3)$

2. Sketch a graph of the continuous even function $g(x)$ over the closed interval of $x$ values $[-5, 5]$ having a range of $g(x)$ values $[-1, 0]$. For $x \geq 0$, roots of $g(x) = 0$ occur at every whole number $k$ and roots of $g'(x) = 0$ occur at $\frac{k}{2}$. The first and second derivatives of $g(x)$ exist everywhere except at $x = k$. Furthermore, $g''(x) > 0$ for every $x \neq k$.

symm to y-axis

cc up



3. Sketch a function $h(x)$ from the following information:

(a) $h(-x) = -h(x)$ → odd : symm to origin

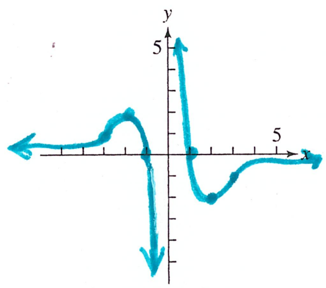
(b) $\lim_{x \to 0^+} h(x) = \infty$ → Vert. Asym

(c) $\lim_{x \to +\infty} h(x) = 0$ → Horz. Asym

(d) For $x > 0$, $h(x) = 0$ only at $x = 1$ → root

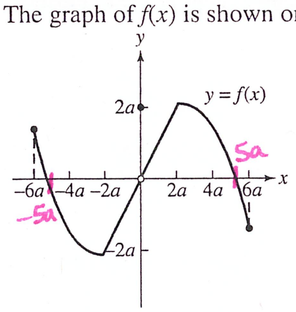
(e) For $x > 0$, $h'(x) = 0$ only at $x = 2$ → min (crt. pt.)

(f) For $x > 0$, $h''(x) = 0$ only at $x = 3$ → pt. of infl.




**Concept Connectors**

4. The graph of $f(x)$ is shown on the closed interval $[-6a, 6a]$:




Answer the following questions regarding $f(x)$:

(a) For $x \neq 0$, the graph of $f(x)$ has symmetry about the origin, that is $f(-x) =$ $-f(x)$.

(b) $f$ has point(s) of discontinuity at $x =$ 0.

(c) $\lim_{x \to 0} f(x) =$ 0.

(d) The zeros of $f(x)$ occur at $x =$ $-5a, 5a$.

(e) $f'(x)$ does not exist at $x =$ $-2a, 0, 2a$ $\pm 6a$?

(f) $f''(x) < 0$ for the $x$ interval(s) $(2a, 6a)$.
ccdown