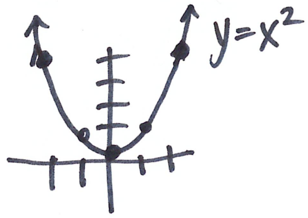
Extreme Values of Functions (Section 5.1)

* Absolute Maximum: If $f(x) \le f(c)$ for all x's in the domain.
Absolute Minimum: If $f(x) \ge f(c)$ for all x's in the domain.
y-values

ex: 

Domain	Absol. Max	Absol Min
$(-\infty,\infty)$	None!	$y=0$
$[0,2]$	$y=4$	$y=0$
$(0,2]$	$y=4$	None!
$[0,2)$	None!	$y=0$
$(0,2)$	None!	None!

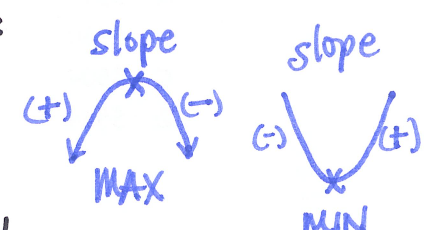
* Extreme Value Theorem: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has BOTH an absolute max and an absolute min.

* Local Max/Min: If $\underbrace{f(x) \le f(c)}_{\text{max}} / \underbrace{f(x) \ge f(c)}_{\text{min}}$ for all x in some open interval containing c.

ex: 

** Critical Points: Any point in the domain where:
1. $f'(x)=0$
2. $f'(x)$ DNE
3. End points on a CLOSED interval

↗ Help us find max/min

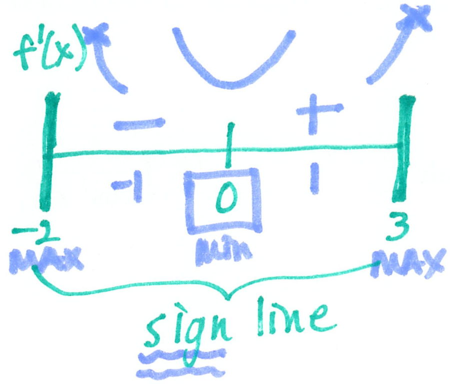


ex: $f(x) = x^{2/3}$ on $[-2,3]$

$f'(x) = \frac{2}{3}x^{-1/3} = \boxed{\frac{2}{3\sqrt[3]{x}}} = 0$

$2 \neq 0$ $\sqrt[3]{x} = 0 \to$ DNE

$\underline{x=0}$
C.P.



$f(0) = 0^{2/3} = $ 0 → Absol. Min

$f(-2) = (-2)^{2/3} = \sqrt[3]{4}$ → Local Max

$f(3) = (3)^{2/3} = \sqrt[3]{9}$ → Absol. Max

ex: $f(x) = \frac{1}{\sqrt{4-x^2}}$ Find Absol Max/Min & Local Max/Min and justify your answers.

$f(x) = (4-x^2)^{-1/2}$ Dom: $(-2,2)$

$4-x^2>0$

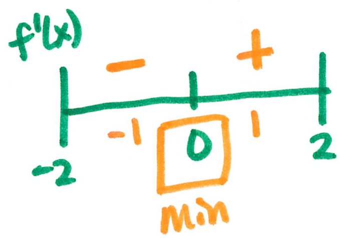
$\sqrt{4} > \sqrt{x^2}$

$2>x$ $-2<x$

$-2<x<2$

$f'(x) = -\frac{1}{2}(4-x^2)^{-3/2}(-2x) = \boxed{\frac{x}{(4-x^2)^{3/2}}} = 0$

$x=0$



$f(0) = \frac{1}{\sqrt{4-0^2}} = \frac{1}{2} \to$ Absolute Min

↳ $y=1/2$ is an absol. min because $f'(x)$ changes from (−) to (+)

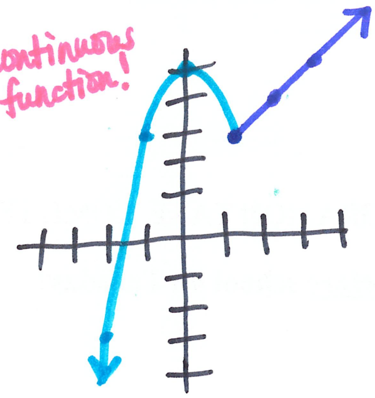
**WHO, WHAT, WHERE

ex: $f(x) = \begin{cases} 5-2x^2 ; & x \le 1 \\ x+2 ; & x > 1 \end{cases}$

x	y
1	3
0	5
-1	3
-2	-3

x	y
1	3
2	4
3	5
4	6

continuous function!



$f'(x) = \begin{cases} -4x ; & x \le 1 \\ 1 ; & x > 1 \end{cases}$

$-4x = 0$

$x = 0$

$f'(x)$: + | − | +

-1 $\boxed{0}$ .1 $\boxed{1}$ 2

Max Min

$f(0) = 5 \rightarrow$ Local Max

$f(1) = 3 \rightarrow$ Local Min

Now... show that $f'(\underline{1})$ DNE.

$$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{(5-2x^2)-3}{x-1} = \lim_{x\to 1^-} \frac{-2x^2+2}{x-1} = \lim_{x\to 1^-} \frac{-2(x^2-1)}{x-1}$$

$$= \lim_{x\to 1^-} \frac{-2(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = -2(1+1) = -4$$

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{(x+2)-3}{x-1} = \lim_{x\to 1^+} \frac{x-1}{x-1} = 1$$

the right & left-hand limits are NOT =

$\therefore f'(1)$ DNE

$$\lim_{h\to 0^+} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0^+} \frac{(x+h+2)-(x+2)}{h} = \lim_{h\to 0^+} \frac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h}$$

$$= \lim_{h\to 0^+} \frac{h}{h} = 1$$