Connecting f' & f'' to the graph of f(x). (Section 5.3)

*** First Derivative Test:**

1. If $f'(x)$ changes from (+) to (−) → Max
2. If $f'(x)$ changes from (−) to (+) → Min
3. If $f'(x)$ does not change sign → No extreme values
4. On a closed interval, you MUST check the end points!

Left: If $f'(x) < 0$: Max
If $f'(x) > 0$: Min

Right: If $f'(x) < 0$: Min
If $f'(x) > 0$: Max

* To determine Absolute/Local: check y-values in f(x).

ex: $f(x) = e^x(x^2-3)$ on $[-5, 3]$ Find all extrema.

$$f'(x) = e^x(2x) + (x^2-3)e^x$$
$$= e^x(x^2+2x-3)$$
$$= e^x(x+3)(x-1) = 0$$

$\cancel{0}$ $x = -3$ $x = 1$

$f'(x)$: + − +

$\boxed{-5}$ (−4) -3 (0) 1 (2) $\boxed{3}$

min max min max

$f(-5) = e^{-5}(25-3) = 22e^{-5}$ → Local Min
$f(-3) = e^{-3}(9-3) = 6e^{-3}$ → Local Max
$f(1) = e^{1}(1-3) = -2e$ → Absol. Min
$f(3) = e^{3}(9-3) = 6e^{3}$ → Absol. Max

*** Second Derivative Test:**

1. If $f''(x) > 0$ → it's concave up ∪, and $f'(x)$ is increasing
2. If $f''(x) < 0$ → it's concave down ∩, and $f'(x)$ is decreasing
3. If $f''(x) = 0$ AND changes concavity, then x is a Point of Inflection, otherwise x is just a C.P.

→ use the calculator to get values!

ex: $s(t) = 2t^3 - 14t^2 + 22t - 5$ $\quad t \geq 0$ where t is a particle moving along a horizontal line.

vel: $s'(t) = 6t^2 - 28t + 22$

*max: $s(1) = 5$

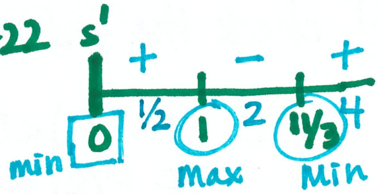
*min: $s(0) = -5$; $s(11/3) = -13.963$

incr → moving Right: $[0,1) \cup (11/3, \infty)$

decr → moving Left: $(1, 11/3)$

$0 = 6t^2 - 28t + 22$

$t = 1, 11/3$



accel: $s''(t) = 12t - 28$

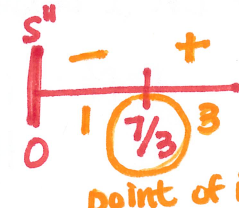
ccup: $(7/3, \infty)$

ccdown: $[0, 7/3)$

*pts. of infl: $s(7/3) = -4.481 \rightarrow (7/3, -4.481)$

$0 = 12t - 28$

$t = 7/3$



* Using $f'(x)$ & $f''(x)$ to graph $f(x)$:

① Identify extrema of $f(x)$ & where they occur

* $f'(x) = 0$ (make a sign line!); $f''(x) = 0$ (max/min of $f'(x)$)

② Identify intervals of incr/decr. & cc up/cc down

③ Sketch!

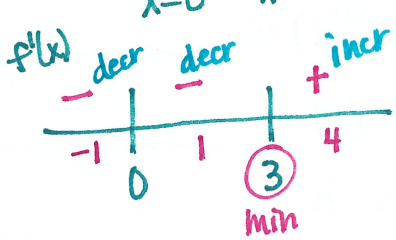
(*Table on pg. 217 is a summary)

ex: $f'(x) = 4x^3 - 12x^2$ Sketch $f(x)$.

$0 = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

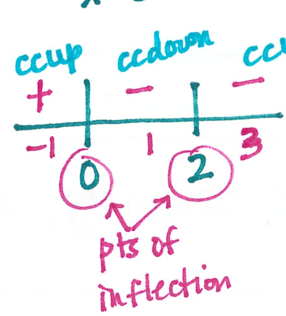
$x = 0 \quad x = 3$



$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

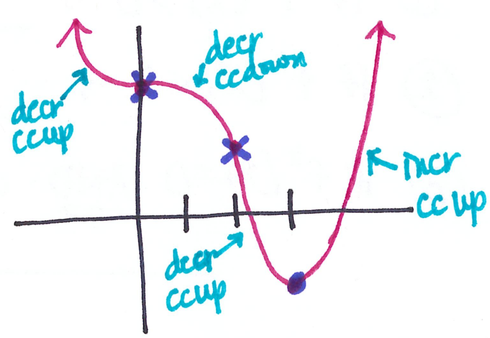
$x = 0 \quad x = 2$



*Make a table w/ ALL critical pts!

$(-\infty, 0)$	$(0, 2)$	$(2, 3)$	$(3, \infty)$
decr	decr	decr	incr
ccup	ccdown	ccup	cc up

x=0 pt of infl; x=2 pt of infl.; x=3 min



For graphing: ① Plot your max/min

② Put on your pts. of infl using incr/decr info!

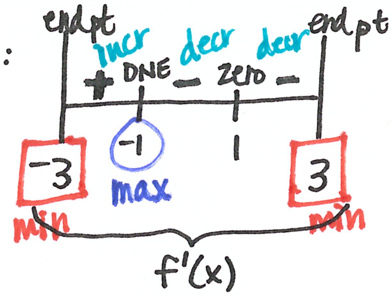
③ Graph the concavity

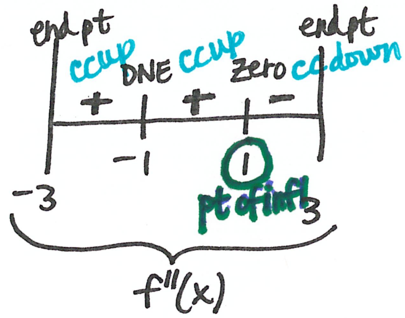
④ Check all parts w/ your table!

* Points of Inflection are an x

* Max/Min are a •

ex: Given:





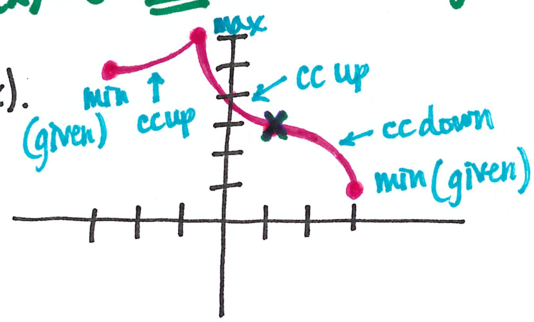
Points:
(−3,5) & (3,1)

a) Absol. Max? @ x=−1
Why? f'(x) changes from (+) to (−)

b) Absol. Min? @ x=3
Why? f(x) is decreasing to that end-pt & (3,1) is lower than (−3,5).

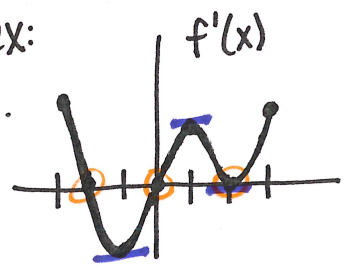
c) Pt. of Infl? @ x=1
Why? f''(x)=0 AND f''(x) changes from (+) to (−).

d) Sketch f(x).



① Plot the points we know (−3,5) & (3,1)
② Plot the max @ x=−1
*put it above (−3,5)
③ Plot the pt. of infl.
*use incr/decr
④ plot concavity.

ex:

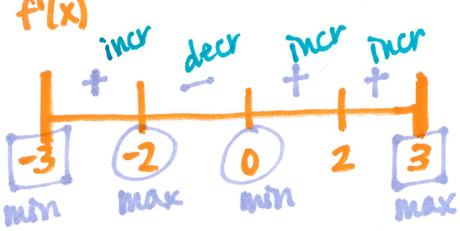


a) Rel Max @ x= −2, 3.
Rel Min @ x= −3, 0.

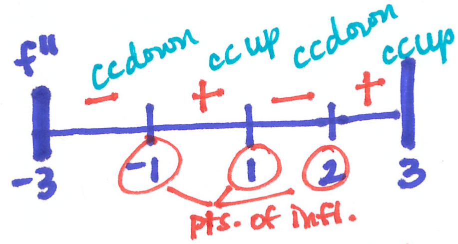
b) cc up: (−1,1) ∪ (2,3]
why? the slope of f'(x) is (+)

f'(x)=0 @ x=−2,0,2
slope of f'(x)=0 @ x=−1,1,2

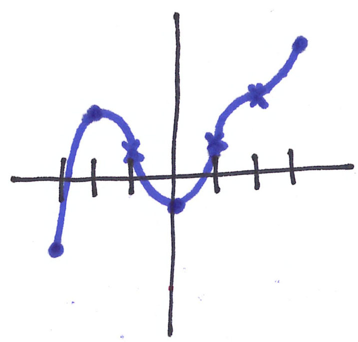
c) Sketch f(x).



*look @ values of f'(x)



* look @ slopes of f'(x).



**I just made x=−3 my absol. min & x=3 my absol. max. We don't know that for sure!