Related Rates (Section 5.6)

* Any equation involving two or more variables that are differentiable functions of time can be used to find related rates.

* Type 1: Geometric Formulas

STRATEGY: ① Write the formula(s).
② Write all given information and the goal.
③ Implicitly differentiate w/respect to time
④ Substitute in given info & Don't forget the units!

ex: When a balloon is inflated, the volume increases w/respect to time and so does the radius. If the radius is increasing at a rate of 2 in/min, find the rate of change of the volume when the radius is 6 in.

① $V = \frac{4\pi}{3}r^3$

② $r = 6\text{ in}$

$\frac{dr}{dt} = 2\text{ in/min}$

$\frac{dV}{dt} = ?$

③ $V = \frac{4\pi}{3}r^3$

$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

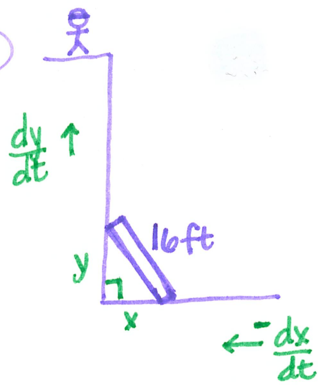
④ $\frac{dV}{dt} = 4\pi(6)^2(2)$

$\boxed{\frac{dV}{dt} = 288\pi \frac{\text{in}^3}{\text{min}}}$

* Type 2: Pythagorean Theorem Applications

STRATEGY: ① Draw a picture! Label all quantities & rates.
② Write given info & the goal.
③ Write an appropriate Pythagorean formula
④ Implicitly differentiate w/respect to time
⑤ Substitute in given values & included units.

ex: A construction worker pulls a 16 ft plank up the side of a building by a rope. Assume the plank follows the rope up the wall of the building. The worker pulls the rope at a rate of 0.5 ft/sec. How fast is the plank sliding along the ground when it is 8 ft from the wall of the building?

① 

② plank = 16ft

$\frac{dy}{dt} = 0.5$ ft/sec

$x = 8$

↓

$8^2 + y^2 = 16^2$

$y = \sqrt{192}$

$\frac{dx}{dt} = ?$

③ $x^2 + y^2 = 16^2$

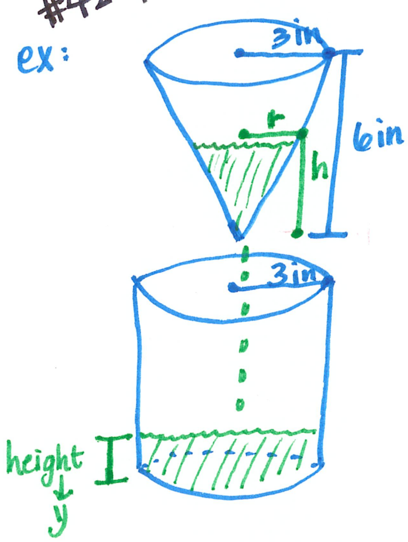
④ $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{-y}{x} \frac{dy}{dt}$

⑤ $\frac{dx}{dt} = \frac{-\sqrt{192}}{8}(0.5)$

$\boxed{\frac{dx}{dt} \approx -0.866 \text{ ft/sec}}$

#42 pg 258

ex: 

Cone

$V = \frac{\pi}{3} r^2 h$

$\frac{dV}{dt} = -10$ in³/min

Cylinder

$V = \pi r^2 h$

$\frac{dV}{dt} = 10$ in³/min

@ h = 5 in.

* radius NOT changing ∴

$V = \pi(3)^2 h$

$V = 9\pi h$ OR

$V = 9\pi y$

a) Find $\frac{dy}{dt}$ @ h = 5 in.

$V = 9\pi y$

$\frac{dV}{dt} = 9\pi \cdot \frac{dy}{dt}$

$10 = 9\pi \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = \frac{10}{9\pi} \approx 0.354 \text{ in/min}}$

b) Find $\frac{dh}{dt}$ @ h = 5 in.

* Use a proportion!

$\frac{3}{r} = \frac{6}{h}$

$3h = 6r$

$r = \frac{1}{2}h$

∴ $V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$ → $-10 = \frac{\pi}{4}(5)^2 \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-8}{5\pi} \approx -0.509 \text{ in/min}}$