DATE________________________

6.2 Concepts Worksheet

NAME________________________

Riemann Sum as a Definite Integral

Certain infinite series can be summed because they are identifiable as the Riemann sum of a definite integral. The following are both Riemann sums. The first example outlines steps in the process of converting a Riemann sum to a definite integral.

Example A:

Find a definite integral whose value is equal to $\lim\limits_{n\to\infty}\left(\dfrac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{\sqrt{n^3}}\right)$.

Step 1: Factor out a "candidate" dx. In general, dx will be a constant divided by n. Note in this case we may use $dx=\dfrac{1}{n}$.

$$\lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{\sqrt{n^3}}\right)=\lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{\sqrt{n}}\right)\cdot\frac{1}{n}$$

$$=\lim_{n\to\infty}\left[\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\cdots+\sqrt{\frac{n}{n}}\right]\cdot\frac{1}{n}$$

Step 2: Identify the variable, x, by searching the summation for changing values where the absolute value of the common difference between consecutive values is dx. (There are several possible appropriate choices for x and dx.)

x-values: $\dfrac{1}{n}, \dfrac{2}{n}, \ldots, \dfrac{n}{n}$ $\quad\left(x=\dfrac{k}{n}\right)$

Step 3: Find the range of x values as n approaches infinity.

Limit of x-values: $\lim\limits_{n\to\infty}\dfrac{1}{n}=0;\ \lim\limits_{n\to\infty}\dfrac{n}{n}=1$

Step 4: Verify that dx is a correct "candidate" for this Riemann Sum. Specifically, dx should equal the quotient determined by the width of the range of x-values divided by the number of terms in the summation. That is, $dx=\dfrac{1-0}{n}$, which agrees with the dx selected in Step 1.

Step 5: Identify $f(x)$, the functional operation on the x-values. Since the summation terms $\sqrt{\dfrac{1}{n}}, \sqrt{\dfrac{2}{n}}, \ldots, \sqrt{\dfrac{n}{n}}$ are the square roots of the x-values: $\dfrac{1}{n}, \dfrac{2}{n}, \ldots, \dfrac{n}{n}$, we let $f(x)=\sqrt{x}$.

In summary, $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\left[\left(\sqrt{\dfrac{k}{n}}\right)\dfrac{1}{n}\right]$ is a Riemann sum, which can be expressed as $\int_0^1\sqrt{n}\,dx$, a definite integral.

6.2 Concepts Worksheet Continued NAME________________

Example B:

Briefly outlined:

$$\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{k^3}{n^4} = \lim_{n\to\infty}\sum_{k=1}^{2n}\left[\frac{k^3}{n^3}\cdot\frac{1}{n}\right]$$

$$= \lim_{n\to\infty}\sum_{k=1}^{2n}\left[\left(\frac{k}{n}\right)^3\cdot\frac{1}{n}\right]$$

Step 1: $dx = \frac{1}{n}$

Step 2: x-values: $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{2n}{n}$ $\left(x = \frac{k}{n}\right)$

Step 3: Limit of x-values: $\lim_{n\to\infty}\frac{1}{n} = 0$; $\lim_{n\to\infty}\frac{2n}{n} = 2$

Step 4: $dx = \frac{2-0}{2n} = \frac{1}{n}$, which agrees with dx chosen in Step 1.

Step 5: $f(x) = x^3$

Therefore $\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{k^3}{n^4} = \int_0^2 x^3\,dx$

Express the following Riemann Sums as definite integrals:

1. $\lim_{n\to\infty}\left[\frac{1}{n^3}+\frac{4}{n^3}+\frac{9}{n^3}+\cdots+\frac{n^2}{n^3}\right]$

2. $\lim_{n\to\infty}\sum_{k=1}^{n}\left[\frac{k}{n}+\left(\frac{k}{n}\right)^2\right]\frac{1}{n}$

3. $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{1}{n^2}\left(1+\frac{2k}{n}\right)}$

4. $\lim_{n\to\infty}\sum_{k=n+1}^{2n}\frac{1}{2k}$

5. $\lim_{n\to\infty}\sum_{k=1}^{2n}\left[\frac{1}{1+\frac{2k}{n}}\cdot\frac{1}{n}\right]$

6.2 Concepts Worksheet Continued NAME______________________________

Concept Connectors

Based on problem 5 above, use the following choices for x-values and define an appropriate integral.

6. $x = \dfrac{k}{n}$

____

7. $x = \dfrac{2k}{n}$

____

8. $x = 1 + \dfrac{2k}{n}$

____

9. Do the integrals you wrote in problems 6–8 all have the same value? Explain.

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DATE______________________

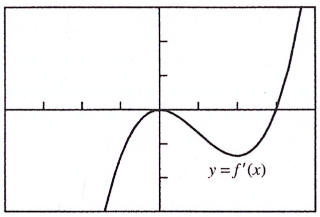
6.3–6.4 Concepts Worksheet

NAME______________________

Graphical Antidifferentiation

Each of the following graphs represents the derivative of a continuous function f. Sketch a possible graph of $y = f(x)$ on the same set of axes as the derivative, assuming $f(0) = 0$.

1.



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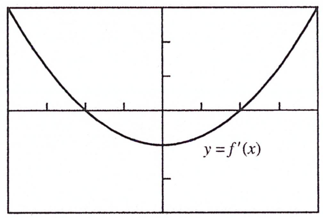

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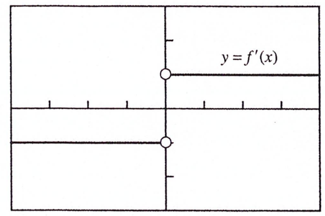
3.



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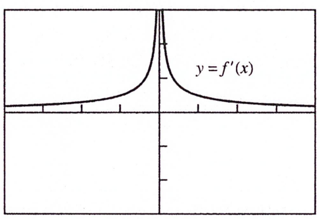
4.



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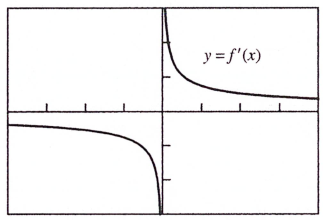
5.



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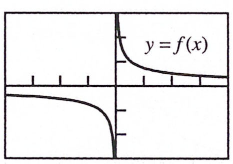
The following graphs of $f'(x)$ involve nonexistent derivatives at $x = 0$, because $f(x)$ is discontinuous at $x = 0$. Match each graph of $f(x)$ to the corresponding $f'(x)$ graph.

6.



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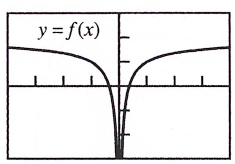
7.



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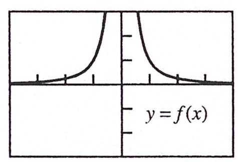

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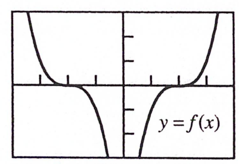
9.



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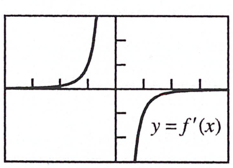
(a)



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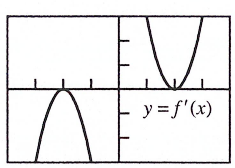
(b)



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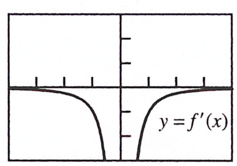
(c)



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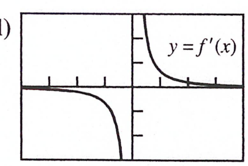
(d)



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