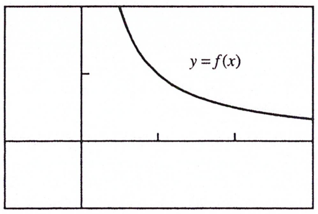
6.3–6.4 Concepts Worksheet Continued NAME________________________________

Given a graph of a function *f*, what would the graph of the function $F(x)=\int_a^x f(t)\,dt$ look like? The following questions should lead you to a rough shape of the graph of $y = F(x)$.

10. Let *f* be the function whose graph is shown below, and let $F(x)=\int_1^x f(t)\,dt,$ for $x>0$:



[−1, 3] by [−1, 2]

(a) Evaluate $F(1)$. ______________________________

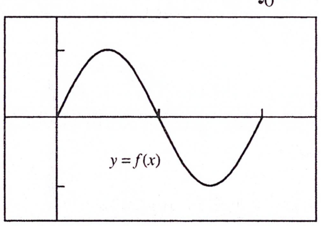
(b) $F(x)$ is always ______________________________
increasing/decreasing

(c) $F(x)$ is negative for what x values?______________________________

(d) State any maximum or minimum points of $F(x)$.

(e) Draw a rough sketch of the integral function, $F(x)$, on the graph above.

11. Let *f* be the function whose graph is shown below, where *f* is defined for $0 \le x \le 2$ and has point symmetry about $(1, 0)$. Let $F(x)=\int_0^x f(t)\,dt$



[−0.5, 2.5] by [−1.5, 1.5]

(a) Evaluate $F(0)$. ______________________________

(b) Evaluate $F(2)$. ______________________________

(c) $F(x)$ has a maximum value at $x =$ ______________________

(d) $F'(x)$ has a maximum value at $x =$ ______________________

(e) $F'(x)$ has a minimum value at $x =$ ______________________

(f) $F''(x)>0$ for what values of x? ______________________

(g) $F''(x)<0$ for what values of x? ______________________

(h) Draw a rough sketch of the integral function, $F(x)$, on the graph above.

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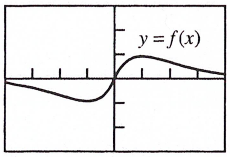
6.3–6.4 Concepts Worksheet Continued   NAME________________________

Concept Connectors

The graph of an odd function f is shown. Let $F(x) = \int_0^x f(t)\,dt$ and assume that $F(a) = b$.



[–4, 4] by [–3, 3]

12. Evaluate each definite integral.

(a) $\int_{-a}^{a} f(t)\,dt$ ______________________________

(b) $\int_{-a}^{a} |f(t)|\,dt$ ______________________________

(c) $\int_{-a}^{a} f(|t|)\,dt$ ______________________________

13. Draw a rough sketch of the integral function $F(x) = \int_0^x f(t)\,dt$ on the same set of axes of the graph of $y = f(x)$.

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6.4 Concepts Worksheet

NAME________________________

The Fundamental Theorem of Calculus

Test your understanding of parts 1 and 2 of the Fundamental Theorem by simplifying the following:

1. If $F(x) = \int_1^x \frac{dt}{1+t^2} dt$, then $F'(x) =$ ________________

2. If $F(x) = \int_3^{2x} \sqrt{t^2+1}\, dt$, then $F'(x) =$ ________________

3. If $F(x) = \int_x^{2x} \frac{1}{t} dt$, $x > 0$, then $F'(x) =$ ________________

4. If $F(x) = \int_0^{\sqrt{x}} \sin(t^2)\, dt$, then $F'(x) =$ ________________

5. If $x > 0$, then $\frac{d}{dx} \int_1^{1/x} \frac{2}{t} dt =$ ________________

6. $\int_1^4 \left(\frac{d}{dx} \sqrt{x^2 - 1} \right) dx =$ ________________

Concept Connectors

Using the definition of a derivative and the Fundamental Theorem of Calculus, evaluate the following:

7. $\lim_{h \to 0} \frac{\int_2^{2+h} \sqrt{x^2 + x}\, dx}{h} =$ ________________

____

8. $\lim_{x \to 1} \frac{\int_1^x \frac{1}{t^4+1} dt}{x - 1} =$ ________________

____

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6.5 Concepts Worksheet

NAME________________________

Definite Integral Approximations

1. Determine and evaluate a definite integral for which $\frac{1}{40}[(0)^3 + 2\cdot(0.05)^3 + 2(0.1)^3 + \cdots + 2(1.95)^3 + (2)^3]$ is a trapezoidal approximation. Which is greater, the integral or trapezoidal approximation? Why?

____

____

2. Determine and evaluate a definite integral for which $\frac{1}{2}\left[\frac{1}{2} + 1.5 + 2 + 2.5 + 3 + \cdots + 9.5 + 5\right]$ is a trapezoidal approximation. Which is greater, the integral or the trapezoidal approximation? Why?

____

____

3. (a) Determine and evaluate a definite integral for which $\frac{1}{6}\left[0 + 4\left(\frac{\sqrt{2}}{2}\right) + 2(1) + 4\left(\frac{\sqrt{6}}{2}\right) + 2\sqrt{2} + 4\left(\frac{\sqrt{10}}{2}\right) + 2\sqrt{3} + 4\left(\frac{\sqrt{14}}{2}\right) + 2\right]$ is a Simpson's rule approximation.

____

(b) Set up the trapezoidal approximation for the integral answer from (a) using the same number of partitions.

____

____

(c) Compare the Simpson's Rule and trapezoidal approximations geometrically. Which do you think should be larger? Why?

____

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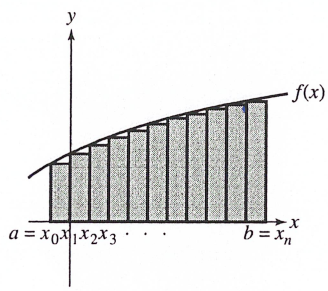
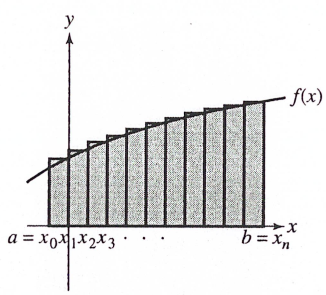
6.5 Concepts Worksheet Continued NAME________________________

Concept Connectors

4. For a strictly increasing positive-valued function over an interval $[a, b]$ explain why the average of the sums of the areas of n inscribed and circumscribed rectangles, all of constant width, is the trapezoidal approximation. (Note: For a strictly decreasing function over an interval, a similar conclusion can be obtained.)



Inscribed Rectangles



Circumscribed Rectangles

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