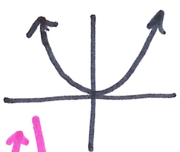
Limits Involving Infinity (Section 2.2)

* When a limit approaches $\pm\infty$, this means you are looking for the END BEHAVIOR of the function.

* End Behavior of Polynomials:

① $y = x^{even}$ ex: $y = x^2$ up/up (↑↑)

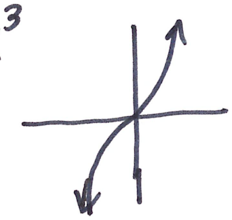


Left: $x \to -\infty$; $y \to +\infty$
Right: $x \to +\infty$; $y \to +\infty$

② $y = -x^{even}$ ex: $y = -x^2$ down/down (↙↓)

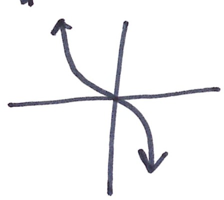
$x \to -\infty$; $y \to -\infty$
$x \to +\infty$; $y \to -\infty$

③ $y = x^{odd}$ ex: $y = x^3$ down/up (↓↑)



$x \to -\infty$; $y \to -\infty$
$x \to +\infty$; $y \to +\infty$

④ $y = -x^{odd}$ ex: $y = -x^3$ up/down (↑↓)



$x \to -\infty$; $y \to +\infty$
$x \to +\infty$; $y \to -\infty$

ex: $y = \boxed{4x^6} - 5x^3 + 7x^2 - 2$

End Beh? (↑↑) $\boxed{\begin{array}{l} x \to -\infty \,;\; y \to +\infty \\ x \to +\infty \,;\; y \to +\infty \end{array}}$

ex: $y = 2x^2 \boxed{-5x^3} + x + 1$

End Beh? (↑↓) $x \to -\infty$; $y \to +\infty$
$x \to +\infty$; $y \to -\infty$

* Vertical Asymptotes: set the denom $= 0$; $x = \_\_$ equations!

* Horizontal Asymptotes: $y = \dfrac{\text{BIG DOG}}{\text{BIG DOG}}$ then reduce!

End Beh. Asym

term w/ highest exponent

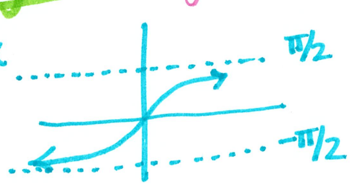
① $y = \dfrac{\#}{\#}$ (All variables have cancelled out)

** ② $y = \dfrac{\#}{\text{vari}} \to y = 0$

③ $y = \dfrac{\text{vari}}{\#} \to$ SLANT ASYM!

you can not have a horz asym AND a slant asym

$y = \arctan x$



can have 2 Horz. Asym

ex: $y = \frac{2x}{x-4}$

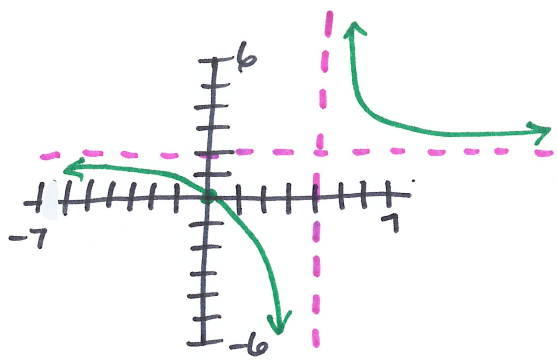
vert asym: $\boxed{x=4}$

horz asym:

$y = \frac{2\cancel{x}}{\cancel{x}} \Rightarrow \boxed{y=2}$

$\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to-\infty} f(x) = 2$



$x=0 \quad y = \frac{2(0)}{0-4} = \frac{0}{-4} = 0$

* Limits w/ infinity: All properties apply: sum, diff, prod, quotient, constant mult...

* Power Rule: $\lim_{x\to\pm\infty} [f(x)]^{r/s} = (L)^{r/s}$

** $\lim_{x\to\infty} \left(\frac{\sin x}{x}\right) = 0$ ← Memorize This!

ex: $\lim_{x\to 2} \sqrt{x+3} = \sqrt{2+3}$ $\boxed{=\sqrt{5}}$

* you could use Power Rule, but its unneccessary!

ex: $\lim_{x\to\infty} \left(\frac{5x+\sin x}{x}\right) = \lim_{x\to\infty} \left(\frac{5\cancel{x}}{\cancel{x}} + \cancel{\frac{\sin x}{x}}^{\,0}\right)$

$= \lim_{x\to\infty} (5+0) \boxed{=5}$

ex: $\lim_{x\to\infty} \left(\frac{x+e^{-x}}{x}\right) = \lim_{x\to\infty} \left(\cancel{\frac{x}{x}}^{\,1} + \frac{e^{-x}}{x}\right) = \lim_{x\to\infty} \left(1 + \frac{e^{-x}}{x}\right) = \lim_{x\to\infty} \left(1 + \cancel{\frac{1}{e^x \cdot x}}^{\,0}\right) = 1+0$ $\boxed{=1}$

* Sandwich Theorem: When functions are bound (upper/lower), you can "sandwich" that function between the bounds.

Arctan x, sin x, cos x

ex: $\lim_{x\to\infty} \left(\frac{1-\cos x}{x^2}\right) \boxed{=0}$

$-1 \le \cos x \le 1$

$\lim_{x\to\infty} \left[\frac{1-(-1)}{x^2}\right] \le \lim_{x\to\infty} \left[\frac{1-\cos x}{x^2}\right] \le \lim_{x\to\infty} \left[\frac{1-(1)}{x^2}\right]$

$\downarrow \qquad\qquad\qquad\qquad \downarrow$

$\lim_{x\to\infty} \left(\frac{2}{x^2}\right) \qquad\qquad \lim_{x\to\infty} \left(\frac{0}{x^2}\right)$

$\downarrow \qquad\qquad\qquad\qquad \downarrow$

$0 \le \lim_{x\to\infty} \left(\frac{1-\cos x}{x^2}\right) \le 0$