Continuity (Section 2.3)

* A continuous function is one that has a y-value for every x-value in the domain.

* Discontinuities: ① Infinite: Asymptotes!

② Jump: Look for these on piece-wise functions.

$$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$$ ← Formal Definition

③ Point (Also called Removable): Hole!

A point discontinuity is when the point, c, has a limit, but the limit does not equal f(c).

$$\lim_{x\to c} f(x) \neq f(c)$$ ← Formal Definition

④ Oscillating: Trig functions.

* Calculus Definition of a Continuous Function:

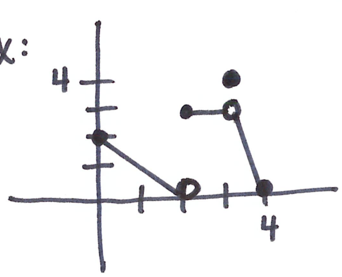
f(x) is continuous @ point c if the left-hand limit and the right-hand limit are equal AND f(c) = limit

$\lim_{x\to c} f(x) = f(c)$ -OR- $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$ ← Formal Definition of an interior point

$$\lim_{x\to a^+} f(x) = f(a)$$ ← Definition of a Left-Endpoint

$$\lim_{x\to b^-} f(x) = f(b)$$ ← Definition of a Right-Endpoint

ex:



Is x=0 continuous? $\lim_{x\to 0^+} f(x) = f(0)$

$2 = 2$ ✓

∴ YES!

Is x=4 continuous? $\lim_{x\to 4^-} f(x) = f(4)$

$0 = 0$ ∴ YES!

Is x=2 continuous?

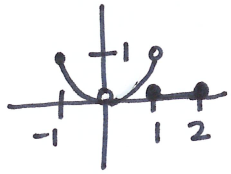
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$0 \neq 1$ ∴ No.

Is x=3 continuous? → $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$1 = 1 \neq 4$ ∴ No.

#23



a) Points of Discon't? $x=0$; $x=1$

b) Which are removable? @ $x=0$ → Hole ∴ removable

@ $x=1$ → Jump ⇒ Not removable.

* Composite Functions: If $f(x)$ is continuous @ point c and $g(x)$ is
($f \circ g$ / $g \circ f$) continuous @ $f(c)$, then $g \circ f$ is con't @ point c.

more general definition { * If the interior function is con't @ point c, and the exterior function is con't @ the y-value of c, then the composite is con't @ point c.

ex: $f(x) = e^x + 3$
$g(x) = \frac{1}{x}$

Is $f \circ g$ con't @ $x=0$? $g(0)$ is not con't ∴ $f \circ g$ is not con't @ $x=0$.

Is $g \circ f$ con't @ $x=0$? $f(0) = e^0 + 3 = 4$
$g(4) = \frac{1}{4}$ ✓ ∴ $g \circ f$ is con't @ $x=0$.

** Intermediate Value Theorem for Continuous Functions:

A function $f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ & $f(b)$. In other words, if y_0 is between $f(a)$ & $f(b)$, then $y_0 = f(c)$ for some value between a & b in $[a,b]$.