Rates of Change / Tangent Lines (Section 2.4)

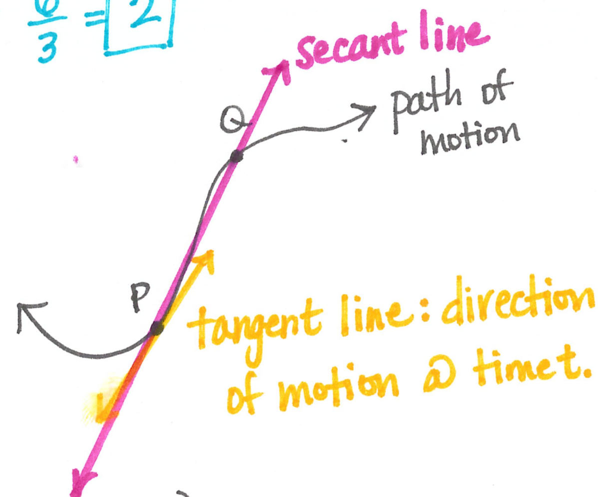
** Average Rate of Change: $m = \frac{\Delta y}{\Delta x}$
(slope)

ex: $f(x) = 2x^2 - 3$ $[-1, 2]$ Find the ave. rate of change.

$$m = \frac{f(2) - f(-1)}{2 + +1} = \frac{5 + +1}{3} = \frac{6}{3} = \boxed{2}$$

* Slope of a line tangent to a curve:
touches at point P

secant line: goes through 2 points



ex: $y = x^2$ @ $P(2,4)$ ⟶ 2^{nd} point: $(2+h,\ f(2+h))$

slope: $$m = \frac{f(2+h) - f(2)}{(2+h) - 2} = \frac{(2+h)^2 - 4}{\cancel{2}+h-\cancel{2}}$$

$$= \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h} = \frac{\cancel{h}(4+h)}{\cancel{h}} = 4+h$$

$$\lim_{h \to 0} (4+h) = 4+0 = \boxed{4 = m}$$

*** Slope of a curve @ Point $(a, f(a))$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

ex: $f(x) = 3 + x^2$ @ $P(-1, 4)$ Find the slope, $\perp$ slope and write an equation of the tangent line for each.

$$m = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0} \frac{[3 + (-1+h)^2] - 4}{h} = \lim_{h \to 0} \frac{\cancel{3} + \cancel{1} - 2h + h^2 - \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2+h)}{\cancel{h}} = \lim_{h \to 0} (-2+h) = -2+0$$

$$\boxed{m = -2} \quad \boxed{\perp m = \frac{1}{2}}$$

$\therefore$ $y - 4 = -2(x+1)$ for slope

$y - 4 = \frac{1}{2}(x+1)$ for $\perp$slope

* equation of tangent line $\rightarrow$ same slope

* equation of normal line $\rightarrow$ $\perp$ slope
($\perp$)

** Speed: instantaneous rate of change of position

$$\lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$