Implicit Differentiation (Section 4.2)

* Steps: (1) Differentiate both sides of the equation w/respect to x.
(2) Collect all terms w/ $\frac{dy}{dx}$ on one side of the equation.
(3) Factor out $\frac{dy}{dx}$.
(4) Solve for $\frac{dy}{dx}$.

ex: $y^2 = x$ Find $\frac{dy}{dx}$.

$$\frac{2y \cdot \frac{dy}{dx}}{2y} = \frac{1 \cdot \frac{dx}{dx}}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2y}}$$

ex: $2y = x^2 + \sin y$ Find $\frac{dy}{dx}$.

$$2\frac{dy}{dx} = 2x\frac{dx}{dx} + \cos y \frac{dy}{dx}$$

$$2\frac{dy}{dx} - \cos y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(2 - \cos y) = 2x$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{2 - \cos y}}$$

ex: $y^2 + 2y = 2x^2 + 1$ Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$.

$$2y\frac{dy}{dx} + 2\frac{dy}{dx} = 4x\frac{dx}{dx}$$

$$\frac{dy}{dx}(2y+2) = 4x$$

$$\frac{dy}{dx} = \frac{4x}{2y+2} = \boxed{\frac{2x}{y+1}}$$

$$\frac{dy}{dx} = \frac{2x}{y+1}$$

$$\frac{d^2y}{dx^2} = \frac{(y+1)(2\frac{dx}{dx}) - 2x(1 \cdot \frac{dy}{dx})}{(y+1)^2}$$

$$= \frac{2(y+1) - 2x\frac{dy}{dx}}{(y+1)^2}$$

$$= \frac{2(y+1)^{(y+1)} - 2x\left(\frac{2x}{y+1}\right)}{(y+1)^{2(y+1)}}$$

$$= \frac{2(y+1)^2 - 4x^2}{(y+1)^3} = \frac{2(y^2+2y+1) - 4x^2}{(y+1)^3}$$

$$\frac{2[(2x^2+1)+1] - 4x^2}{(y+1)^3}$$

$$= \frac{4x^2 + 4 - 4x^2}{(y+1)^3} = \boxed{\frac{4}{(y+1)^3}}$$

ex: $x^2 - xy + y^2 = 7$ Find the tangent & normal line @ $(-1,2)$ point

slope: $2x - x\left(\frac{dy}{dx}\right) + y(-1) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2y - x) = y - 2x$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}$ @ $(-1,2) \rightarrow m = \frac{2+2(+1)}{2(2)+(+1)} = \frac{4}{5}$ $\perp m = -\frac{5}{4}$

Tangent: $y-2 = \frac{4}{5}(x+1)$

Normal: $y-2 = -\frac{5}{4}(x+1)$

ex: $3x^2 - 2xy^2 + 4y = 7$ Find $\frac{dy}{dx}$.

$6x - 2x\left(2y\frac{dy}{dx}\right) + y^2(-2) + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}(4 - 4xy) = 2y^2 - 6x$

$\frac{dy}{dx} = \frac{2y^2-6x}{4-4xy} = \frac{y^2-3x}{2-2xy}$

#44 If $g''(t) = \frac{1}{t^{3/4}}$, Find: a) $g'(t)$ b) $g'''(t)$ c) $g(t)$ Easiest

$g''(t) = t^{-3/4}$

$g'''(t) = -\frac{3}{4}t^{-7/4}$

*To go backwards:

1. Add 1 to the exponent
2. Divide by that exponent
3. Add a constant

a) $-\frac{3}{4} + 1 = \frac{1}{4}$ $g'(t) = \frac{t^{1/4}}{1/4} + C_1$

$= 4t^{1/4} + C_1$

c) $\frac{1}{4} + 1 = \frac{5}{4}$ $g(t) = \frac{4 \cdot t^{5/4}}{5/4} + C_1 t + C_2$

$g(t) = \frac{16}{5}t^{5/4} + C_1 t + C_2$

#46 $y^2(2-x) = x^3$ Find the tangent & normal line @ $(1,1)$.

$y^2(-1) + (2-x)\left(2y\frac{dy}{dx}\right) = 3x^2$

$-y^2 + 4y\frac{dy}{dx} - 2xy\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx}(4y - 2xy) = 3x^2 + y^2$

$\frac{dy}{dx} = \frac{3x^2+y^2}{4y-2xy}$ @ $(1,1) \rightarrow m = \frac{3+1}{4-2} = \frac{4}{2} = 2$ $\perp m = -\frac{1}{2}$

Tangent: $y-1 = 2(x-1)$

Normal: $y-1 = -\frac{1}{2}(x-1)$