Inverse Trig Function Derivatives (Section 4.3)

* $y = \sin^{-1} x$ does <u>NOT</u> equal $y = \frac{1}{\sin x}$

* Derivatives: If $y = \sin^{-1} u$, then $y' = \frac{1}{\sqrt{1-u^2}} \cdot du/dx$

  If $y = \tan^{-1} u$, then $y' = \frac{1}{1+u^2} \cdot du/dx$

- Because there is not a "NICE" derivative for inverse cosine, we use: $\cos^{-1} u = \frac{\pi}{2} - \sin^{-1} u$ <u>BEFORE</u> taking a derivative.
- Add in: $y = \sec^{-1} u \rightarrow y' = \frac{1}{|u|\sqrt{u^2-1}} \cdot du/dx$; $|u| > 1$

ex: $\underbrace{y = \sin^{-1}(x^2)}$ Find $\frac{dy}{dx}$.

$y = \sin^{-1}(u)$ $u = x^2$

$\frac{du}{dx} = 2x$

$y' = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$y' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \boxed{\frac{2x}{\sqrt{1-x^4}}}$

ex: $\underbrace{y = \tan^{-1}(3x-5)}$

$y = \tan^{-1} u$ $u = 3x-5$

$\frac{du}{dx} = 3$

$y' = \frac{1}{1+u^2} \cdot du/dx$

$y' = \frac{1}{1+(3x-5)^2} \cdot 3 = \boxed{\frac{3}{9x^2-30x+26}}$

$(3x-5)^2 \downarrow 9x^2-30x+25$

ex: $y = \cos^{-1}(5x^3) \rightarrow y = \frac{\pi}{2} - \sin^{-1}(5x^3)$

$u = 5x^3$

$\frac{du}{dx} = 15x^2$

$y = \frac{\pi}{2} - \sin^{-1} u$

$y' = \frac{-1}{\sqrt{1-u^2}} \cdot du/dx$

$y' = \frac{-1}{\sqrt{1-(5x^3)^2}} \cdot 15x^2 = \boxed{\frac{-15x^2}{\sqrt{1-25x^6}}}$

#3 $y = \sin^{-1}(\sqrt{2}t)$

$u = \sqrt{2}t$

$\frac{du}{dt} = \sqrt{2}$

$y = \sin^{-1} u$

$y' = \frac{1}{\sqrt{1-u^2}} \cdot du/dt$

$y' = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2} = \boxed{\frac{\sqrt{2}}{\sqrt{1-2t^2}}}$

#6 $\underbrace{y = s(1-s^2)^{1/2} + \cos^{-1}s}$

$y = s(1-s^2)^{1/2} + \frac{\pi}{2} - \sin^{-1}s$

$y' = s\left[\cancel{\tfrac{1}{2}}(1-s^2)^{-1/2}(-\cancel{2}s)\right] + (1-s^2)^{1/2}(1) + \frac{-1}{\sqrt{1-s^2}}\cdot 1$

$y' = \frac{-s^2}{(1-s^2)^{1/2}} + \frac{(1-s^2)^{1/2}\cdot(1-s^2)^{1/2}}{(1-s^2)^{1/2}} + \frac{-1}{(1-s^2)^{1/2}}$

$y' = \frac{-s^2 + \cancel{(1}-s^2)\cancel{-1}}{(1-s^2)^{1/2}} = \boxed{\frac{-2s^2}{(1-s^2)^{1/2}}}$

#8 $y = \frac{1}{\sin^{-1}(2x)}$

$y = (\sin^{-1}2x)^{-1}$

$y' = -1(\sin^{-1}2x)^{-2}\cdot\frac{1}{\sqrt{1-(2x)^2}}\cdot 2$

$\boxed{y' = \frac{-2}{(\sin^{-1}2x)^2(1-4x^2)^{1/2}}}$

#7 $y = x\sin^{-1}x + (1-x^2)^{1/2}$

$y' = x\left[\frac{1}{\sqrt{1-x^2}}\cdot 1\right] + \sin^{-1}x\,(1) + \cancel{\tfrac{1}{2}}(1-x^2)^{-1/2}(-\cancel{2}x)$

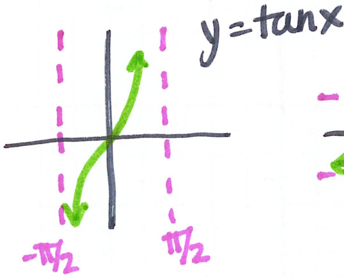
$= \cancel{\frac{x}{(1-x^2)^{1/2}}} + \sin^{-1}x + \cancel{\frac{-x}{(1-x^2)^{1/2}}}$

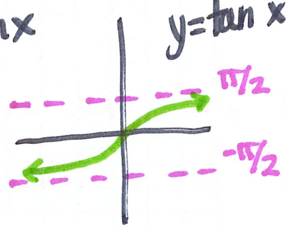
$\boxed{= \sin^{-1}x}$

#27 If $y = \tan^{-1}x$, find: a) right end behavior: $\lim_{x\to+\infty} f(x) = \pi/2$

b) left end behavior: $\lim_{x\to-\infty} f(x) = -\pi/2$

c) any $\underbrace{\text{horz. tangents}}_{m=0}$ → Horz Tangents: NONE





$y = \tan^{-1}x$

$y' = \frac{1}{1+x^2} \stackrel{?}{=} 0$

Never Happens!

4.2 Concept WS

1. $x^5 + x^4 y - xy^2 - y^3 = 0$ Find $\frac{dy}{dx}$.

$$5x^4 + x^4\left(\frac{dy}{dx}\right) + y\left(4x^3\right) - x\left(2y\frac{dy}{dx}\right) + y^2(-1) - 3y^2\frac{dy}{dx} = 0$$

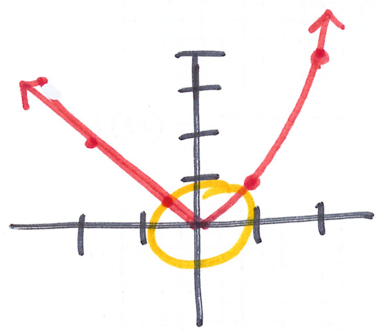
$$\frac{dy}{dx}\left(x^4 - 2xy - 3y^2\right) = y^2 - 5x^4 - 4x^3y$$

$$\boxed{\frac{dy}{dx} = \frac{y^2 - 5x^4 - 4x^3y}{x^4 - 2xy - 3y^2}}$$

2. $\boxed{\left(-\tfrac{1}{2}, \tfrac{1}{2}\right) \; ; \; (-2, 2) \; ; \; (2, 4)}$

$y=-x$; $y=-x$; $y=x^2$

a) $\frac{dy}{dx} = \frac{(\frac{1}{2})^2 - 5(-\frac{1}{2})^4 - 4(-\frac{1}{2})^3(\frac{1}{2})}{(-\frac{1}{2})^4 - 2(-\frac{1}{2})(\frac{1}{2}) - 3(\frac{1}{2})^2} = \frac{0.1875}{-0.1875} = -1$



3. Factored form: $(x+y)(x^2+y)(x^2-y)$

$x+y=0$; $\cancel{x^2+y=0}$; $x^2-y=0$

$y=-x$ (line) ; $\cancel{y=-x^2}$; $y=x^2$ (parabola)

$y'=-1$; No points that fit this! ; $y'=2x$

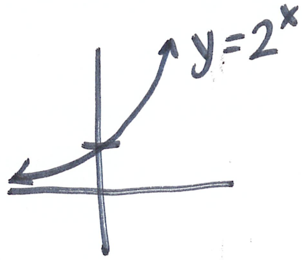
4. Find $\frac{dy}{dx}$ @ $(0,0)$? No! $\frac{dy}{dx}$ is undefined @ $(0,0)$

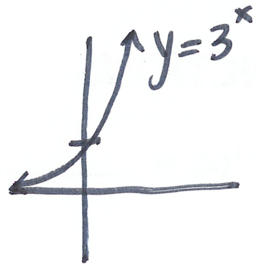
5. $\lim\limits_{x \to 0^+} \left[\frac{f(x) - f(0)}{x - 0}\right] = 2(0) = 0$

6. $\lim\limits_{x \to 0^-} \left[\frac{f(x) - f(0)}{x - 0}\right] = -1$

7. Evaluate $f'(0)$? No, the left hand & right hand limit are NOT equal.

#8





What "special" number lies between 2 & 3?

$e \approx 2.714\ldots$

So... $f(x)=e^x$ then $f'(x)=e^x$

4.3 Concept WS

#1-4 Graph the inverse. (Symmetric to $y=x$ line)

#5 Inverse Function? (Pass the vertical AND horizontal line test)

#7 $\boxed{f(g(x)) = g(f(x)) = x}$ → Definition of Inverses! $f(x)$ & $g(x)$ must be inverses!

#8 $g'(x)$ expression.

$$f(g(x)) = x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

#9-12

x	f(x)	f'(x)
1	-3	1/2
2	-2	2
3	1	4

x	g(x)	g'(x)
-3	1	2
-2	2	—
1	3	—

$$g'(-3) = \frac{1}{f'(g(-3))} = \frac{1}{f'(1)} = \frac{1}{1/2} = 2$$

#10 tangent of $f(x)$ @ $x=1$

Point: $(1,-3)$

Slope: $\frac{1}{2}$

$$\boxed{y+3=\tfrac{1}{2}(x-1)}$$