Derivatives of Exponential & Log Functions (Section 4.4)

* If $y=e^u$, then $y'=e^u\cdot\frac{du}{dx}$
* If $y=u^a$, then $y'=au^{a-1}\cdot\frac{du}{dx}$ (power rule)
* If $y=a^u$, then $y'=a^u\cdot\ln a\cdot\frac{du}{dx}$
exponential function
* If $y=\log_a u$, then $y'=\frac{1}{u\cdot\ln a}\cdot\frac{du}{dx}$
* If $y=\ln u$, then $y'=\frac{1}{u}\cdot\frac{du}{dx}$

* "u" is a function of x
"v" is a function of x
"a" is a constant.

** If $y=u^v$, then
$$y'=y\left[\frac{v}{u}\cdot\frac{du}{dx}+\ln u\cdot\frac{dv}{dx}\right]$$
OR
① Take the "ln" of both sides of the equation.
② Use implicit differentiation to solve for $\frac{dy}{dx}$.

ex: $y=e^{4x-5}$
$y'=e^{4x-5}\cdot(4)=\boxed{4e^{4x-5}}$

ex: $y=x^{1-e}$
$y'=(1-e)x^{\cancel{1}-e\cancel{-1}}$
$\boxed{y'=(1-e)x^{-e} \text{ OR } \frac{1-e}{x^e}}$

ex: $y=x^2e^x-xe^x=e^x(x^2-x)$
$y'=e^x(2x-1)+(x^2-x)e^x$
$\boxed{y'=e^x[x^2+x-1]}$

#9 on WS
ex: $y=\ln^2x=(\ln x)^2$
$y'=2(\ln x)^1\left(\frac{1}{x}\right)=\boxed{\frac{2\ln x}{x}}$

#16 on WS
ex: $y=\log_5\sqrt{x}=\log_5 x^{1/2}=\frac{1}{2}\log_5 x$
$y'=\frac{1}{\sqrt{x}\ln 5}\left(\frac{1}{2}x^{-1/2}\right)$
$\boxed{y'=\frac{1}{2x\ln 5}}$ ✓
$y'=\frac{1}{2}\left(\frac{1}{x\ln 5}\right)$
$\boxed{y'=\frac{1}{2x\ln 5}}$

ex: $y=x^{1/\ln x}$ ① Take the ln of both sides
$\ln y=\ln x^{1/\ln x}$
$\ln y=\frac{1}{\cancel{\ln x}}\cdot\cancel{\ln x}$
$\ln y=1$ ② Implicit
$\cancel{y}\cdot\frac{1}{\cancel{y}}\frac{dy}{dx}=0\cdot y$
$\boxed{\frac{dy}{dx}=0}$
OR $\ln y=1$
$e^1=y$
$\boxed{y'=0}$ ✓

#24 on WS
ex: $y=\ln 10^x=x\ln 10$
$\boxed{y'=\ln 10}$

ex: $y = x \cdot e^x$ Find the Normal equation of a line @ $\underline{(0,0)}$.

$y' = x(e^x) + e^x(1)$

$= e^x(x+1)$ @ $(0,0)$

$y' = e^0(0+1) = 1$

$m = 1$

$\perp m = -1$

$y - 0 = -1(x-0)$

$\boxed{y = -x}$

ex: Prove if $y = \ln(k \cdot x)$, then $y' = \frac{1}{x}$ for <u>any</u> constant k.

$y = \ln k + \ln x$

$y' = 0 + \frac{1}{x}$

$y' = \frac{1}{x}$ ✓

$y' = \frac{1}{\cancel{k} \cdot x} \cdot \cancel{k}$

$y' = \frac{1}{x}$ ✓

ex: $y = x^{\tan x}$

$\ln y = \tan x \ln x$

$\frac{1}{y} \cdot \frac{dy}{dx} = \tan x \left(\frac{1}{x}\right) + \ln x (\sec^2 x)$

$\frac{dy}{dx} = y\left[\frac{\tan x}{x} + \ln x (\sec^2 x)\right]$

$$\boxed{\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \ln x (\sec^2 x)\right]}$$