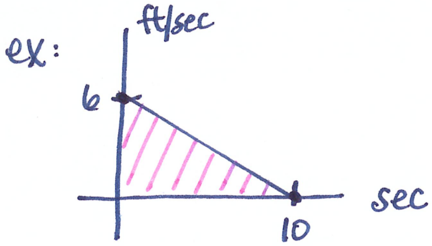
Estimating with Finite Sums (Section 6.1) DAY 1

* Distance traveled = Area Under the Curve

ex:



$d = r \cdot t = (6\ \text{ft/sec})(10\ \text{sec}) = 60\ \text{ft}$ but you didn't go 6 ft/sec the whole time!

so...

distance traveled = area under the curve → Triangle

$= \frac{1}{2} b \cdot h$

$= \frac{1}{2}(10)(6) = \boxed{30\ \text{ft}}$

* Area Under the Curve can be found by Rectangular Approximations:

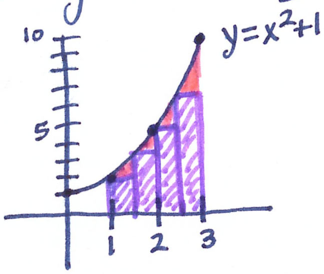
LRAM, RRAM, MRAM ← Rectangular Approximation Method (RAM)

Left, Right, Mid-point

$A = b \cdot h = \Delta x \cdot h$

$\Delta x = \dfrac{b-a}{n}$ ← upper − lower limits; n ← # of rectangles

ex: $y = x^2+1$ [1,3] 4 Rectangles. Find LRAM, RRAM, & MRAM.



LRAM: $\Delta x = \dfrac{b-a}{n} = \dfrac{3-1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

$h \cdot \Delta x$

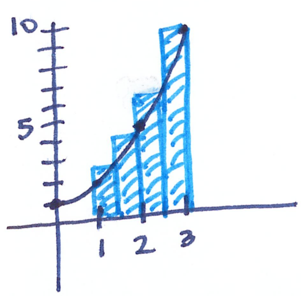
$\text{LRAM} = \frac{1}{2} \cdot f(1) + \frac{1}{2} \cdot f(1.5) + \frac{1}{2} f(2) + \frac{1}{2} \cdot f(2.5)$

$= \frac{1}{2}(2) + \frac{1}{2}(3.25) + \frac{1}{2}(5) + \frac{1}{2}(7.25)$

$= \frac{1}{2}[2 + 3.25 + 5 + 7.25]$

$= \frac{1}{2}(17.5) \boxed{= 8.75}$ → underestimate!

* Left corner touches the graph!

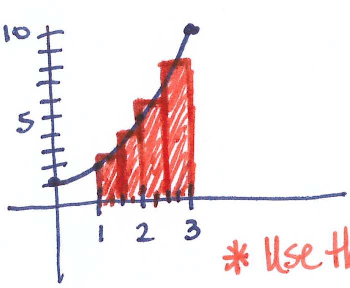


$\text{RRAM} = \frac{1}{2} \cdot f(1.5) + \frac{1}{2} f(2) + \frac{1}{2} f(2.5) + \frac{1}{2} f(3)$

$= \frac{1}{2}(3.25) + \frac{1}{2}(5) + \frac{1}{2}(7.25) + \frac{1}{2}(10)$

$= \frac{1}{2}[3.25 + 5 + 7.25 + 10] = \frac{1}{2}(25.5) \boxed{= 12.75}$ overestimate!

* Right corner touches the graph!



$\text{MRAM} = \frac{1}{2} \cdot f(1.25) + \frac{1}{2} f(1.75) + \frac{1}{2} f(2.25) + \frac{1}{2} f(2.75)$

$= \frac{1}{2}(2.5625) + \frac{1}{2}(4.0625) + \frac{1}{2}(6.0625) + \frac{1}{2}(8.5625)$

$= \frac{1}{2}[2.5625 + 4.0625 + 6.0625 + 8.5625]$

$= \frac{1}{2}(21.25) = \boxed{10.625}$

* Use the mid-point of the left & right height.

4 2 2 3

ex:

x	-1	3	5	7	10
y	2	1	3	5	6

Find LRAM & RRAM using 4 rectangles.

same widths!

LRAM $= 4(2) + 2(1) + 2(3) + 3(5)$
$= 8 + 2 + 6 + 15$
$= 31$

RRAM $= 4(1) + 2(3) + 2(5) + 3(6)$
$= 4 + 6 + 10 + 18$
$= 38$

MRAM = Inappropriate! No x-value midpts to get y-values!

2 2

ex:

x	0	1	2	3	4
y	2	6	4	5	8

Find LRAM, RRAM & MRAM using 2 rectangles.

you have mid-pt x-values to get y-values!

LRAM $= 2(2) + 2(4) = 12$

RRAM $= 2(4) + 2(8) = 24$

MRAM $= 2(6) + 2(5) = 22$

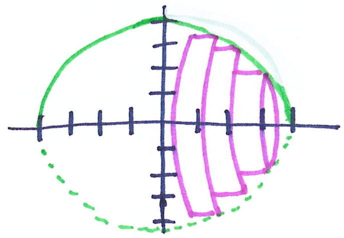
* Total Volume = Sum of All Sliced Volumes (Section 6.1) **Day 2**

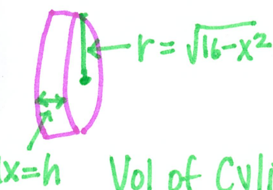
ex: $f(x) = \sqrt{16-x^2}$ revolved about the x-axis

↳ This makes a sphere w/radius of 4.

$V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3}(4)^3$

$\boxed{V = 268.083 \text{ units}^3}$





Vol of Cylinder $= \pi r^2 h = \pi(\sqrt{16-x^2})^2(\Delta x)$

$= \pi(16-x^2)\Delta x$

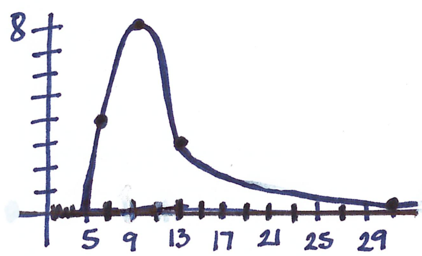
* If $\Delta x = 1$, then the sum of the volumes of the cylinders is:

$$V = \pi \underbrace{\left[(16-(-4)^2)+(16-(-3)^2)+(16-(-2)^2)+(16-(-1)^2)+(16-0^2)+(16-1^2)+(16-2^2)+(16-3^2)\right]}_{\text{sums of } 16-x^2}\overset{\Delta x}{(1)}$$

LRAM (but an RRAM would be the same value!)

$$V = \pi[0+7+12+15+16+15+12+7] = \boxed{84\pi \approx 263.894 \text{ units}^3}$$

*Cardiac Output: Table 6.2 on pg. 272



LRAM: $A = b \cdot h \cdot 2[f(5)+f(7)+f(9)+\ldots+f(29)]$

$\boxed{= 55.1}$

RRAM $= 2[f(7)+f(9)+f(11)+\ldots f(31)] = \boxed{55.1}$

MRAM $= 2[f(6)+f(8)+f(10)+\ldots f(30)] =$???

Not appropriate because f(6), f(8)... are NOT on the table of values

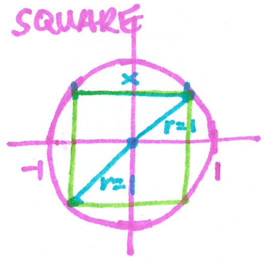
Ex #4 on pg. 273

$$\text{Cardiac Output} = \frac{\text{amount of dye}}{\text{area under the curve}} \times \frac{60 \text{ sec}}{1 \text{ min}}$$

$$= \frac{5.6}{55.1} \times \frac{60}{1} = \boxed{6.10 \text{ L/min}}$$

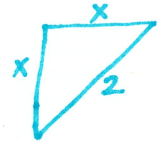
#37 (Work in groups)

a) SQUARE



$x^2+y^2=r^2$

$y=\sqrt{1-x^2}$



$x^2+x^2=2^2$

$2x^2=4$

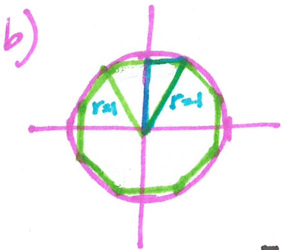
$x=\sqrt{2}$

$A=x^2$

$A=(\sqrt{2})^2$

$\boxed{A=2}$

b)



OCTAGON

$A=16(\text{area of } \triangle)$

$\theta=\frac{2\pi}{16}=\frac{\pi}{8}$

Area of Triangle $=\frac{1}{2}\sin\theta\cdot\cos\theta$

$A=16\left[\frac{1}{2}\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right]$

$\boxed{A\approx 2.828}$

c) 16-sided-gon $\rightarrow$ $A=32(\text{area of } \triangle)$

$\theta=\frac{2\pi}{32}=\frac{\pi}{16}$

$A=32\left[\frac{1}{2}\sin\frac{\pi}{16}\cdot\cos\frac{\pi}{16}\right]$

$\boxed{A\approx 3.061}$

d) Area of a circle $=\pi r^2$

$A=\pi(1)^2$

$\boxed{\begin{array}{c} A\approx 3.142 \\ \therefore \text{ as } n\to\infty\text{, the Area}\to\pi \end{array}}$