# Definite Integrals (Section 6.2)

* Riemann Sums: Think "RECTANGLES" → LRAM, RRAM, MRAM

All are examples of Riemann Sums

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

$$S_n = \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k$$

* Definite Integral of a continuous function on $[a,b]$:

Let $f$ be continuous on $[a,b]$ and $[a,b]$ is partitioned into "n" subintervals of equal length so that $\Delta x = \frac{b-a}{n}$.

Then the definite integral of $f$ over $[a,b]$ is:

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k) \cdot \Delta x$$

* Symbols & Terms:

$\int_a^b f(x)\,dx$

$b$ ← upper limit

$a$ ← lower limit

integration sign

variable of integration

integrand: the function being integrated

"The integral of $f(x)$ from $a$ to $b$."

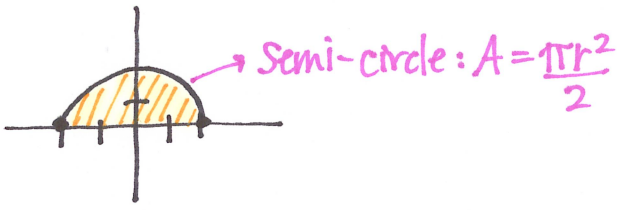
Finds the signed area (+/−) under the curve.

ex: Evaluate: $\int_{-2}^{2} \sqrt{4-x^2}\,dx$

Area under the curve

$= \frac{\pi(2)^2}{2}$

$= 2\pi$

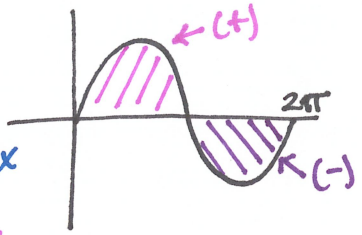



* If the function is below the x-axis → The area will be (−).

* If the function has both (−) and (+) areas, you must seperate them if you are finding TOTAL area.

ex: TOTAL area of $\int_a^b f(x)\,dx$ = (Area ↑ x-axis) − (Area ↓ x-axis)

ex: Find the TOTAL area of $\int_0^{2\pi} \sin x\,dx$.




$\int_0^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx$ OR $2\int_0^{\pi} \sin x\,dx$

$2 - (-2) = 4$ $\quad 2(2) = 4$

How do we find this?

Use the calc: MATH, #9, fnINT → $(\sin x, x, 0, \pi) = 2$

* If only asked to evaluate: $\int_0^{2\pi} \sin x\,dx$ **= 0**

On Calc: ex: $\int_{-1}^{2} x \cdot \sin x\,dx = 2.043$

ex: $\int_0^{1} \frac{4}{1+x^2}\,dx = 3.142 \to \pi$

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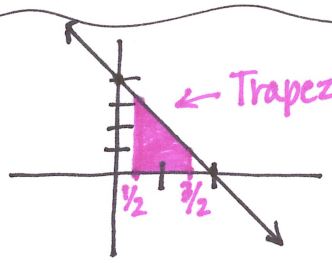
ex: $\int_{1/2}^{3/2} (-2x+4)\,dx$ **= 2**





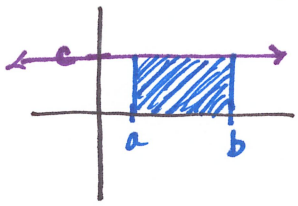
$A = \frac{1}{2}h(a+b)$

$A = \frac{1}{2}(1)(1+3)$

$A = 2$

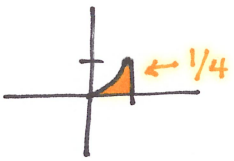
* Integration of a constant:

If $f(x) = c$ on the interval $[a,b]$, then $\int_a^b f(x)\,dx = \int_a^b c\,dx = (b-a)c$

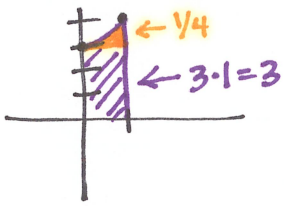



Area: $l \cdot w$ → $(b-a)\,c$

* Like #47-55 on HW: $\int_0^{1} x^3\,dx = \frac{1}{4}$




ex: Find $\int_0^{1} (x^3+3)\,dx$ **= 3.25**




ex: Find $\int_{-1}^{1} |x|^3\,dx$ **= $\frac{2}{4}$ or $\frac{1}{2}$**