Chapter 2 Review: 1–49 odd

1. $\lim_{x\to -2} x^3-2x^2+1 = -8-8+1 = \boxed{-15}$

3. $\lim_{x\to 4} \sqrt{1-2x} = \sqrt{1-8} = \sqrt{-7} \to \boxed{\text{DNE}}$

5. $\lim_{x\to 0} \dfrac{\frac{1}{2+x}-\frac{1}{2}}{x} = \lim_{x\to 0} \dfrac{\frac{\cancel{2}-\cancel{2}-x}{2(2+x)}}{x} = \lim_{x\to 0} \dfrac{\frac{-\cancel{x}}{4+2x}}{\cancel{x}} = \lim_{x\to 0} \dfrac{-1}{4+2x} = \dfrac{-1}{4+0} = \boxed{\dfrac{-1}{4}}$

7. $\lim_{x\to \infty} \dfrac{x^4+x^3}{12x^3+128} = \lim_{x\to \infty} \dfrac{x^4}{12x^3} = \lim_{x\to \infty} \dfrac{x}{12} = \dfrac{\infty}{12} = \boxed{\infty}$

$\lim_{x\to -\infty} \dfrac{x^4+x^3}{12x^3+128} = \lim_{x\to -\infty} \dfrac{x^4}{12x^3} = \lim_{x\to -\infty} \dfrac{x}{12} = \dfrac{-\infty}{12} = \boxed{-\infty}$

9. $\lim_{x\to 0} \dfrac{x\csc x+1}{x\csc x} = \lim_{x\to 0} \dfrac{x\csc x}{x\csc x} + \dfrac{1}{x\csc x} = \lim_{x\to 0} 1+\dfrac{\sin x}{x} = 1+1 = \boxed{2}$

11. $\lim_{x\to 3.5^+} \text{int}(2x-1) = \text{int}(2\cdot 3.5-1) = \text{int}(6) = \boxed{6}$

13. $\lim_{x\to \infty} e^{-x}\cos x = \lim_{x\to \infty} \dfrac{\cos x}{e^x} = \dfrac{\cos\infty}{e^\infty} = \dfrac{\pm 1}{\infty} = \boxed{0}$

15. Limit exists

17. Limit exists

19. Limit exists

21. Continuous

23. Discontinuous

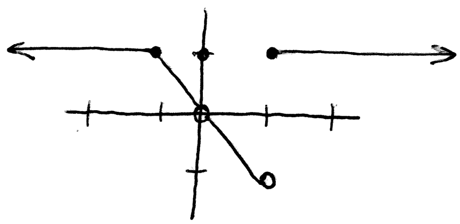
25. a) 1
    b) 1.5
    c) No
    d) $x=3$ (point), $x<-1$ (domain), $x>3$ (domain)
    e) Removable at $x=3$ with $f(3)=1$.

27. $f(x) = \dfrac{x+3}{x+2}$

a) $x+2=0 \to \boxed{x=-2}$

b) $\lim_{x\to -2^+} f(x) = \boxed{\infty}$ $\lim_{x\to -2^-} f(x) = \boxed{-\infty}$

29. $f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$



a) $\lim_{x\to -1^-} f(x) = 1$, $\lim_{x\to -1^+} f(x) = 1$ b) Yes, 1 c) Yes, limit = point

$\lim_{x\to 0^-} f(x) = 0$, $\lim_{x\to 0^+} f(x) = 0$ Yes, 0 No, limit $\ne$ point

$\lim_{x\to 1^-} f(x) = -1$, $\lim_{x\to 1^+} f(x) = 1$ No, left $\ne$ right No, limit DNE

31. $f(x) = \dfrac{x+1}{4-x^2}$ $4-x^2=0 \to x^2=4 \to \boxed{x=2,\ x=-2}$

33. $f(x) = \dfrac{2x+1}{x^2-2x+1} \to \dfrac{2x}{x^2} = \boxed{\dfrac{2}{x}}$ $\lim_{x\to\infty} \dfrac{2}{x} = \dfrac{2}{\infty} = 0 \to$ HA at $\boxed{y=0}$

35. $f(x) = \dfrac{x^3-4x^2+3x+3}{x-3} \to \dfrac{x^3}{x} = \boxed{x^2}$ $\lim_{x\to\infty} x^2 = \infty^2 \to \boxed{\text{No HA}}$

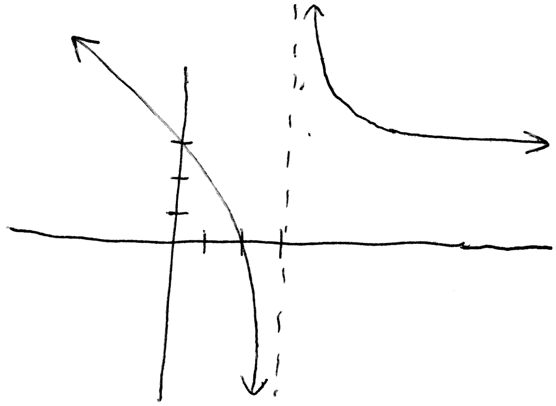
37. $f(x) = x + e^x$

Right: $\lim_{x\to\infty} x+e^x = \infty + e^{\infty} \to \boxed{e^x}$

Left: $\lim_{x\to -\infty} x+e^x = -\infty + e^{-\infty} = -\infty + \cancel{\dfrac{1}{e^{\infty}}} = -\infty + 0 \to \boxed{x}$

39. $\dfrac{(x+5)\cancel{(x-3)}}{\cancel{x-3}} = x+5$ at $x=3 \to 3+5 = \boxed{8}$

41.



43. $f(x) = 1 + \sin x \quad [0, \pi/2]$

$f(\pi/2) = 1 + \sin \pi/2 = 1 + 1 = 2$

$f(0) = 1 + \sin 0 = 1 + 0 = 1$

$\dfrac{2-1}{\pi/2 - 0} = \dfrac{1}{\pi/2} = \boxed{\dfrac{2}{\pi}}$

45. $S = 6x^2, \; x = a$

$\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h\to 0} \dfrac{6(a+h)^2 - 6a^2}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{6a^2} + 12ah + 6h^2 - \cancel{6a^2}}{h}$

$\lim\limits_{h\to 0} 12a + 6h = 12a + 0 = \boxed{12a}$

47. $f(x) = x^2 - 3x$

$f(1) = 1^2 - 3(1) = -2 \rightarrow (1, -2)$

a) $\lim\limits_{h\to 0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h\to 0} \dfrac{(1+h)^2 - 3(1+h) - 1^2 + 3\cdot 1}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{1} + 2h + h^2 - \cancel{3} - 3h - \cancel{1} + \cancel{3}}{h}$

$\lim\limits_{h\to 0} \dfrac{h^2 - h}{h} = \lim\limits_{h\to 0} h - 1 = 0 - 1 = \boxed{-1}$

b) $y + 2 = -1(x - 1) \rightarrow y + 2 = -x + 1 \rightarrow \boxed{y = -x - 1}$

c) $m = 1 \rightarrow y + 2 = 1(x - 1) \rightarrow y + 2 = x - 1 \rightarrow \boxed{y = x - 3}$

49. $p(t) = \dfrac{200}{1+7e^{-0.1t}} = \dfrac{200}{1+\frac{7}{e^{0.1t}}}$

a) $p(0) = \dfrac{200}{1+\frac{7}{e^0}} = \dfrac{200}{1+\frac{7}{1}} = \dfrac{200}{8} = \boxed{25}$

There were 25 bears when the wildlife preserve was created.

b) $\lim\limits_{t\to\infty} p(t) = \dfrac{200}{1+\frac{7}{e^{\infty}}} = \dfrac{200}{1+0} = \dfrac{200}{1} = \boxed{200}$

c) As time goes on, the bear population approaches 200. The carrying capacity of number of bears in the wildlife preserve is 200.