Chain Rule Practice with Trig Functions - Key

1. $f(x) = \sin x \cot x = \sin x \cdot \dfrac{\cos x}{\sin x} = \cos x$

$f'(x) = \boxed{-\sin x}$

2. $f(x) = \dfrac{\tan x}{1+x^2}$

$f'(x) = \dfrac{(1+x^2)\sec^2 x + {}^-\tan x(2x)}{(1+x^2)^2} = \boxed{\dfrac{(1+x^2)\sec^2 x - 2x\tan x}{(1+x^2)^2}}$

3. $g(w) = \dfrac{1+\sec w}{1-\sec w}$

$g'(w) = \dfrac{(1-\sec w)(\sec w \tan w) - (1+\sec w)(-\sec w \tan w)}{(1-\sec w)^2}$

$g'(w) = \dfrac{\sec w \tan w - \cancel{\sec^2 w \tan w} + \sec w \tan w + \cancel{\sec^2 w \tan w}}{(1-\sec w)^2}$

$g'(w) = \boxed{\dfrac{2\sec w \tan w}{(1-\sec w)^2}}$

4. $k(v) = \dfrac{\csc v}{\sec v} = \dfrac{(1/\sin v)}{(1/\cos v)} = \dfrac{1}{\sin v} \cdot \dfrac{\cos v}{1} = \dfrac{\cos v}{\sin v} = \cot v$

$k'(v) = \boxed{-\csc^2 v}$

5. $k(x) = \sin(x^2+2)$

$k'(x) = \cos(x^2+2) \cdot 2x = \boxed{2x\cos(x^2+2)}$

6. $H(x) = \cos^5 3x = (\cos 3x)^5$

$H'(x) = 5(\cos 3x)^4 \cdot -\sin 3x \cdot 3 = \boxed{-15\cos^4 3x \sin 3x}$

7. $g(x) = \sin^4(x^3) = (\sin x^3)^4$

$g'(x) = 4(\sin x^3)^3 \cdot \cos x^3 \cdot 3x^2 = \boxed{12x^2 \sin^3 x^3 \cos x^3}$

8. $t(z) = \sec(2z+1)^2$

$t'(z) = \sec(2z+1)^2 \cdot \tan(2z+1)^2 \cdot 2(2z+1) \cdot 2$

$t'(z) = \boxed{(8z+4)\sec(2z+1)^2 \tan(2z+1)^2}$

9. $f(x) = \dfrac{\sec 2x}{1+\tan 2x}$

$$f'(x) = \frac{(1+\tan 2x)(\sec 2x \tan 2x \cdot 2) - (\sec 2x)(\sec^2 2x \cdot 2)}{(1+\tan 2x)^2}$$

$$f'(x) = \frac{2\sec 2x \tan 2x + 2\sec 2x \tan^2 2x - 2\sec^3 2x}{(1+\tan 2x)^2}$$

$\tan^2 x + 1 = \sec^2 x$

so

$\tan^2 x - \sec^2 x = -1$

$$f'(x) = \frac{2\sec 2x(\tan 2x + \tan^2 2x - \sec^2 2x)}{(1+\tan 2x)^2}$$

$$f'(x) = \boxed{\frac{2\sec 2x(\tan 2x - 1)}{(1+\tan 2x)^2}}$$

10. $F(x) = \dfrac{\cos 4x}{1-\sin 4x}$

$$F'(x) = \frac{(1-\sin 4x)(-4\sin 4x) - (\cos 4x)(-4\cos 4x)}{(1-\sin 4x)^2}$$

$$F'(x) = \frac{-4\sin 4x + 4\sin^2 4x + 4\cos^2 4x}{(1-\sin 4x)^2}$$

$\sin^2 x + \cos^2 x = 1$

so

$4\sin^2 4x + 4\cos^2 4x = 4$

$$F'(x) = \frac{-4\sin 4x + 4}{(1-\sin 4x)^2} = \frac{4(1-\cancel{\sin 4x})}{(1-\sin 4x)^{\cancel{2}}} = \frac{4}{(1-\sin 4x)^1} = \boxed{\frac{4}{1-\sin 4x}}$$

11. $r(a) = \csc(a^2+4)$

$r'(a) = -\csc(a^2+4) \cdot \cot(a^2+4) \cdot 2a = \boxed{-2a\csc(a^2+4)\cot(a^2+4)}$

12. $H(s) = \cot(s^3-2s)$

$H'(s) = -\csc^2(s^3-2s)(3s^2-2) = \boxed{(2-3s^2)\csc^2(s^3-2s)}$

13. $f(x) = \tan(2x^2+3)$

$f'(x) = \sec^2(2x^2+3) \cdot 4x = \boxed{4x\sec^2(2x^2+3)}$

14. $f(x) = \cos(3x^2) + \cos^2(3x)$

$f'(x) = -\sin(3x^2) \cdot 6x + 2\cos(3x) \cdot -\sin(3x) \cdot 3$

$f'(x) = \boxed{-6x\sin(3x^2) - 6\cos(3x)\sin(3x)}$

15. $g(w) = \tan^3 6w = (\tan 6w)^3$

$g'(w) = 3\tan^2 6w \cdot \sec^2 6w \cdot 6 = \boxed{18\tan^2 6w \sec^2 6w}$

16. $F(t) = \csc^2 2t = (\csc 2t)^2$

$F'(t) = 2\csc 2t \cdot -\csc 2t \cdot \cot 2t \cdot 2 = \boxed{-4\csc^2 2t \cot 2t}$

17. $M(x) = \sec\left(\frac{1}{x^2}\right) = \sec(x^{-2})$

$M'(x) = \sec\left(\frac{1}{x^2}\right) \cdot \tan\left(\frac{1}{x^2}\right) \cdot -2x^{-3} = \boxed{\frac{-2}{x^3}\sec\left(\frac{1}{x^2}\right)\tan\left(\frac{1}{x^2}\right)}$

18. $k(z) = z^2 \cdot \cot 5z$

$k'(z) = z^2 \cdot -\csc^2 5z \cdot 5 + \cot 5z \cdot 2z = \boxed{-5z^2\csc^2 5z + 2z\cot 5z}$

19. $H(x) = x \cdot \csc(x^2)$

$H'(x) = x \cdot -\csc(x^2) \cdot \cot(x^2) \cdot 2x + \csc(x^2) \cdot 1$

$H'(x) = \boxed{-2x^2\csc(x^2)\cot(x^2) + \csc(x^2)}$

20. $L(x) = \tan^2 x \cdot \sec^3 x = (\tan x)^2 \cdot (\sec x)^3$

$L'(x) = \tan^2 x \cdot 3\sec^2 x \cdot \sec x \cdot \tan x + \sec^3 x \cdot 2\tan x \cdot \sec^2 x$

$L'(x) = \boxed{3\sec^3 x\tan^3 x + 2\sec^5 x\tan x}$

21. $H(u) = u^2 \cdot \sec^3 4u$

$H'(u) = u^2 \cdot 3\sec^2 4u \cdot \sec 4u \cdot \tan 4u \cdot 4 + \sec^3 4u \cdot 2u$

$H'(u) = 12u^2\sec^3 4u\tan 4u + 2u\sec^3 4u$

$H'(u) = \boxed{2u\sec^3 4u\,(6u\tan 4u + 1)}$

22. $N(x) = (\sin 5x - \cos 5x)^5$

$N'(x) = 5(\sin 5x - \cos 5x)^4 (5\cos 5x + 5\sin 5x)$

$N'(x) = \boxed{(25\sin 5x + 25\cos 5x)(\sin 5x - \cos 5x)^4}$

23. $P(v) = \sin 4v \cdot \csc 4v = \cancel{\sin 4v} \cdot \frac{1}{\cancel{\sin 4v}} = 1$

$P'(v) = \boxed{0}$

24. $f(x) = \sin\sqrt{x} + \sqrt{\sin x} = \sin x^{1/2} + (\sin x)^{1/2}$

$f'(x) = \cos x^{1/2} \cdot \frac{1}{2} x^{-1/2} + \frac{1}{2}(\sin x)^{-1/2} \cdot \cos x$

$f'(x) = \boxed{\frac{\cos\sqrt{x}}{2\sqrt{x}} + \frac{\cos x}{2\sqrt{\sin x}}}$

25. $t(x) = (\tan 2x - \sec 2x)^3$

$t'(x) = 3(\tan 2x - \sec 2x)^2 (2\sec^2 2x - 2\sec 2x \tan 2x)$

$t'(x) = 6\sec 2x(\sec 2x - \tan 2x)(\tan 2x - \sec 2x)^2$

$t'(x) = 6\sec 2x \cdot -1(\tan 2x - \sec 2x)(\tan 2x - \sec 2x)^2$

$t'(x) = \boxed{-6\sec 2x(\tan 2x - \sec 2x)^3}$

26. $f(x) = \tan\sqrt[3]{5-6x} = \tan(5-6x)^{1/3}$

$f'(x) = \sec^2(5-6x)^{1/3} \cdot \frac{1}{3}(5-6x)^{-2/3} \cdot -6$

$f'(x) = \sec^2(5-6x)^{1/3} \cdot -2(5-6x)^{-2/3}$

$f'(x) = \boxed{\dfrac{-2\sec^2\sqrt[3]{5-6x}}{(5-6x)^{2/3}}}$

27. $f(t) = \cos(4-3t)$

$f'(t) = -\sin(4-3t) \cdot -3 = \boxed{3\sin(4-3t)}$