Chain Rule Practice w/ Trig Functions

1. $f(x) = \sin x \cdot \cot x = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$

$$f'(x) = -\sin x$$

2. $f(x) = \frac{\tan x}{1+x^2}$ $\quad f'(x) = \frac{(1+x^2)(\sec^2 x) - (\tan x)(2x)}{(1+x^2)^2}$

3. $f(x) = \frac{1+\sec w}{1-\sec w}$ $\quad f'(x) = \frac{(1-\sec w)(\sec w \cdot \tan w) + (1+\sec w)(+\sec w \cdot \tan w)}{(1-\sec w)^2}$

$$= \frac{\sec w \tan w - \sec^2 w \tan w + \sec w \tan w + \sec^2 w \tan w}{(1-\sec w)^2}$$

$$= \frac{2 \sec w \tan w}{(1-\sec w)^2}$$

4. $f(x) = \frac{\csc v}{\sec v} = \frac{\frac{1}{\sin v}}{\frac{1}{\cos v}} = \frac{\cos v}{\sin v} = \cot v$ $\quad f'(x) = -\csc^2 v$

5. $K(x) = \sin(x^2+2)$ $\quad K'(x) = \cos(x^2+2) \cdot 2x$

$$= 2x \cos(x^2+2)$$

6. $H(x) = \cos^5(3x)$ $\quad H'(x) = 5[\cos(3x)]^4 \cdot [\sin(3x) \cdot 3]$

$$= -15\cos^4(3x)\sin(3x)$$

7. $g(x) = \sin^4(x^3)$ $\quad g'(x) = 4[\sin(x^3)]^3[\cos(x^3) \cdot 3x^2]$

$$= 12x^2\sin^3(x^3)\cos(x^3)$$

8. $t(z) = \sec(2z+1)^2$ $\quad t'(z) = \sec(2z+1)^2 \tan(2z+1)^2 \cdot [2(2z+1)'(2)]$

$$= (8z+4)\sec(2z+1)^2\tan(2z+1)^2$$

9. $f(x) = \dfrac{\sec 2x}{1+\tan 2x}$

$$f'(x) = \frac{(1+\tan 2x)(\sec 2x \tan 2x)(2) - \sec 2x(\sec^2 2x)(2)}{(1+\tan 2x)^2}$$

10. $F(x) = \dfrac{\cos 4x}{1-\sin 4x}$ $\quad F'(x) = \dfrac{(1-\sin 4x)(-\sin 4x)(4) - \cos 4x(-\cos 4x)(4)}{(1-\sin 4x)^2}$

$$= \frac{-4\sin 4x + \overbrace{4\sin^2 4x + 4\cos^2 4x}^{=4}}{(1-\sin 4x)^2}$$

$$= \frac{-4\sin 4x + 4}{(1-\sin 4x)^2}$$

11. $r(a) = \csc(a^2+4)$ $\quad r'(a) = -\csc(a^2+4)\cdot\cot(a^2+4)(2a)$

$$= -2a\csc(a^2+4)\cot(a^2+4)$$

12. $H(s) = \cot(s^3 - 2s)$ $\quad H'(s) = -\csc^2(s^3-2s)\cdot(3s^2-2)$

$$= (2-3s^2)\csc^2(s^3-2s)$$

13. $f(x) = \tan(2x^2+3)$ $\quad f'(x) = \sec^2(2x^2+3)\cdot 4x$

$$= 4x\sec^2(2x^2+3)$$

14. $f(x) = \cos(3x^2) + \cos^2(3x)$ $\quad f'(x) = -\sin(3x^2)\cdot(6x) + 2(\cos 3x)(-\sin 3x)\cdot 3$

$$= -6x\sin(3x^2) - 6\cos 3x \sin 3x$$

15. $g(w) = \tan^3 6w$ $\quad g'(w) = 3(\tan 6w)^2(\sec^2 6w)\cdot 6$

$$= 18\tan^2 6w \sec^2 6w$$

16. $F(t) = \csc^2 2t$ $\quad F'(t) = 2(\csc 2t)^1(-\csc 2t \cot 2t)(2)$

$$= -4\csc^2 2t \cot 2t$$

17. $M(x) = \sec(x^{-2})$ $\quad M'(x) = \sec x^{-2} \tan x^{-2}\cdot -2x^{-3}$

$$= -2x^{-3}\sec(x^{-2})\tan(x^{-2})$$

18. $K(z) = z^2 \cot 5z$ $\quad K'(z) = z^2(-\csc^2 5z)(5) + \cot 5z\,(2z)$

$$= -5z^2\csc^2 5z + 2z\cot 5z$$

19. $H(x) = x \csc x^2$

$H'(x) = x\left(-\csc x^2 \cot x^2\right)(2x) + \csc x^2 (1)$

$= -2x^2 \csc x^2 \cot x^2 + \csc x^2$

$\boxed{= \csc x^2 \left(-2x^2 \cot x^2 + 1\right)}$

20. $L(x) = \tan^2 x \sec^3 x$

$L'(x) = \tan^2 x \left[3(\sec x)^2 (\sec x \tan x)\right] +$

$\sec^3 x \left[2(\tan x)'(\sec^2 x)\right]$

$\boxed{= 3\tan^3 x \sec^3 x + 2\tan x \sec^5 x}$

21. $H(u) = u^2 \sec^3 4u$

$H'(u) = u^2\left[3(\sec 4u)^2 (\sec 4u \tan 4u)\cdot 4\right] +$

$\sec^3 4u\,(2u)$

$= 12u^2 \sec^3 4u \tan 4u + 2u \sec^3 4u$

$\boxed{= 2u \sec^4 4u \left(6u \tan 4u + 1\right)}$

22. $N(x) = (\sin 5x - \cos 5x)^5$

$N'(x) = 5(\sin 5x - \cos 5x)^4 \left[\cos 5x(5) + \sin 5x(5)\right]$

$\boxed{= 25(\sin 5x - \cos 5x)^4 (\cos 5x + \sin 5x)}$

23. $P(v) = \sin 4v \csc 4v = \sin 4v \cdot \frac{1}{\sin 4v} = 1$ $\boxed{P'(v) = 0}$

OR

$P'(v) = \underbrace{\sin 4v}_{=1}\left(-\csc 4v \cot 4v\right)\cdot 4 + \underbrace{\csc 4v}_{=\cot 4v}\left(\cos 4v\right)\cdot 4$

$= -4\cot 4v + 4\cot 4v$

$= 0$

24. $f(x) = \sin x^{1/2} + (\sin x)^{1/2}$ $\quad f'(x) = \cos x^{1/2}\left[\frac{1}{2}x^{-1/2}\right] + \left[\frac{1}{2}(\sin x)^{-1/2}\cdot\cos x\right]$

$$\boxed{f'(x) = \frac{\cos x^{1/2}}{2x^{1/2}} + \frac{\cos x}{2(\sin x)^{1/2}}}$$

25. $t(x) = (\tan 2x - \sec 2x)^3$ $\quad t'(x) = \left[3(\tan 2x - \sec 2x)^2 \cdot (\sec^2 2x \cdot 2 - \sec 2x \tan 2x \cdot 2)\right.$

$$= 6(\tan 2x - \sec 2x)^2(\sec^2 2x - \sec 2x \tan 2x)$$

$$= 6\sec 2x\,(\tan 2x - \sec 2x)^2(\sec 2x - \tan 2x)$$

$$\boxed{= -6\sec 2x\,(\tan 2x - \sec 2x)^3}$$

26. $f(x) = \tan(5-6x)^{1/3}$ $\quad f'(x) = \sec^2(5-6x)^{1/3}\cdot\left[\frac{1}{3}(5-6x)^{-2/3}(-6)\right]$

$$\boxed{= \frac{-2\sec^2(5-6x)^{1/3}}{(5-6x)^{2/3}}}$$

27. $f(t) = \cos(4-3t)$ $\quad f'(t) = -\sin(4-3t)\cdot(-3)$

$$\boxed{= 3\sin(4-3t)}$$