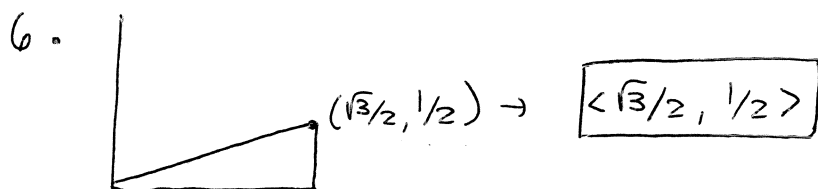


## Ch. 11 Review: 2-46 e.o.o., 48

$$2. \langle -3+2, 4-5 \rangle = \boxed{\langle -1, -1 \rangle}$$

$$\sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \boxed{\sqrt{2}}$$



$$10. x = 1 + t^{-2}, y = 1 - 3t^{-1}, t = 2$$

$$a) x = 1 + \frac{1}{2^2} = 1 + 1/4 = 5/4$$

$$y = 1 - \frac{3}{2} = \frac{2}{2} - \frac{3}{2} = -1/2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3/t^2}{-2/t^3} \Big|_{t=2} = \frac{3/4}{-2/8} = \frac{3/4}{-1/4} = -3$$

$$y + 1/2 = -3(x - 5/4) \rightarrow y + 1/2 = -3x + 15/4 \rightarrow \boxed{y = -3x + 13/4}$$

$$b) \frac{dy}{dx} = \frac{3/t^2}{-2/t^3} = \frac{3}{-2} \cdot \frac{-t^3}{t^2} = \frac{-3}{2} t$$

$$\frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dx/dt} = \frac{-3/2}{-2/t^3} = \frac{-3/2}{-2} \cdot \frac{-t^3}{2} = \frac{3}{4} t^3 \Big|_{t=2} = \frac{3}{4} \cdot 8 = \boxed{6}$$

$$14. x = 4 \cos t, y = 9 \sin t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{9 \cos t}{-4 \sin t}$$

$$a) \text{ Horizontal when } 9 \cos t = 0 \rightarrow \cos t = 0 \rightarrow t = \pi/2, 3\pi/2$$

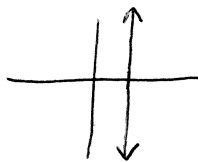
$$\left. \begin{array}{l} t = \pi/2: x = 0, y = 9 \\ t = 3\pi/2: x = 0, y = -9 \end{array} \right\} \boxed{(0, 9), (0, -9)}$$

$$b) \text{ Vertical when } -4 \sin t = 0 \rightarrow \sin t = 0 \rightarrow t = 0, t = \pi$$

$$\left. \begin{array}{l} t = 0: x = 4, y = 0 \\ t = \pi: x = -4, y = 0 \end{array} \right\} \boxed{(4, 0), (-4, 0)}$$

$$18. r \cos \theta = 1 \rightarrow r = \frac{1}{\cos \theta} = \sec \theta$$

$$r \cos \theta = x, \text{ so } x = 1 \rightarrow \boxed{\text{Line}}$$



$$22. r = 2 + \cos 2\theta$$

$$x = r \cos \theta = (2 + \cos 2\theta) \cos \theta$$

$$y = r \sin \theta = (2 + \cos 2\theta) \sin \theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(2 + \cos 2\theta) \cos \theta + \sin \theta (-2 \sin 2\theta)}{(2 + \cos 2\theta)(-\sin \theta) + \cos \theta (-2 \sin 2\theta)} \Big|_{\theta = \pi/3} = \boxed{0.346}$$

$$26. r = 1 + \sin \theta \text{ crosses } x\text{-axis at } \theta = 0, \theta = \pi$$

$$x = r \cos \theta = (1 + \sin \theta) \cos \theta = \cos \theta + \cos \theta \sin \theta$$

$$y = r \sin \theta = (1 + \sin \theta) \sin \theta = \sin \theta + \sin^2 \theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos \theta + 2 \sin \theta \cos \theta}{-\sin \theta + \cos^2 \theta - \sin^2 \theta} \Big|_{\theta=0} = \frac{1 + 2 \cdot 0 \cdot 1}{0 + 1^2 - 0^2} = \frac{1}{1} = 1$$

$$x = \cos 0 + \cos 0 \sin 0 = 1 + 1 \cdot 0 = 1 + 0 = 1 \quad y - 0 = 1(x - 1)$$

$$y = \sin 0 + \sin^2 0 = 0 + 0^2 = 0 \quad \boxed{y = x - 1}$$

$$\frac{\cos \theta + 2 \sin \theta \cos \theta}{-\sin \theta + \cos^2 \theta - \sin^2 \theta} \Big|_{\theta=\pi} = \frac{-1 + 2 \cdot 0 \cdot -1}{0 + (-1)^2 - 0^2} = \frac{-1}{1} = -1$$

$$x = -1 + -1 \cdot 0 = -1 + 0 = -1 \quad y - 0 = -1(x + 1)$$

$$y = 0 + 0^2 = 0 \quad \boxed{y = -x - 1}$$

$$30. r \cos(\theta + \pi/3) = 2\sqrt{3}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \quad (\text{angle sum formula})$$

$$r \cos \theta \cos \pi/3 - r \sin \theta \sin \pi/3 = 2\sqrt{3}$$

$$x \cdot \frac{1}{2} - y \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$\frac{\sqrt{3}}{2} y = \frac{1}{2} x - 2\sqrt{3}$$

$$\sqrt{3} y = x - 4\sqrt{3}$$

$$\boxed{y = \frac{1}{\sqrt{3}} x - 4} \rightarrow \boxed{\text{Line}}$$

$$34. (x+2)^2 + (y-5)^2 = 16$$

$$x^2 + 2x + 2x + 4 + y^2 - 5y - 5y + 25 = 16$$

$$x^2 + 4x + y^2 - 10y = -13$$

$$r^2 \cos^2 \theta + 4r \cos \theta + r^2 \sin^2 \theta - 10r \sin \theta = -13$$

$$r^2 (\underbrace{\cos^2 \theta + \sin^2 \theta}_1) + 4r \cos \theta - 10r \sin \theta = -13$$

$$\boxed{r^2 + 4r \cos \theta - 10r \sin \theta = -13}$$

38. Inside  $r = 2 + 2\sin \theta$  and Outside  $r = 2\sin \theta$   
Cardioid - Circle

$$\frac{1}{2} \int_a^b r^2 d\theta$$

$$\frac{1}{2} \int_0^{2\pi} (2 + 2\sin \theta)^2 d\theta - \frac{1}{2} \int_0^{\pi} (2\sin \theta)^2 d\theta$$

$$\frac{1}{2} \int_0^{2\pi} (4 + 8\sin \theta + 4\sin^2 \theta) d\theta - \frac{1}{2} \int_0^{\pi} 4\sin^2 \theta d\theta$$

$$\int_0^{2\pi} (2 + 4\sin \theta + 2\sin^2 \theta) d\theta - \int_0^{\pi} 2\sin^2 \theta d\theta$$

$\downarrow$   
 $(\frac{1}{2} - \frac{1}{2} \cos 2\theta)$

$$\int_0^{2\pi} (2 + 4\sin \theta + 1 - \cos 2\theta) d\theta - \int_0^{\pi} (1 - \cos 2\theta) d\theta$$

$$\left[ 3\theta - 4\cos \theta - \frac{1}{2} \sin 2\theta \right]_0^{2\pi} - \left[ \theta - \frac{1}{2} \sin 2\theta \right]_0^{\pi}$$

$$(6\pi - 4) - (-4) - (\pi - 0) - (0 - 0) = 6\pi - \cancel{4} + \cancel{4} - \pi = \boxed{5\pi}$$

$$42. \mathbf{r}(t) = \langle e^t \cos t, e^t \sin t \rangle$$

$$\mathbf{v}(t) = \langle -e^t \sin t + e^t \cos t, e^t \cos t + e^t \sin t \rangle$$

$$\mathbf{a}(t) = \langle -e^t \cos t - e^t \sin t + -e^t \sin t + e^t \cos t, -\sin t e^t + \cos t e^t + e^t \cos t + \sin t e^t \rangle$$

$$\mathbf{a}(t) = \langle -2e^t \sin t, 2e^t \cos t \rangle$$

$$\text{Slope of } \mathbf{r}(t) = \frac{y}{x} = \frac{e^t \sin t}{e^t \cos t} = \tan t$$

$$\text{Slope of } \mathbf{a}(t) = \frac{y}{x} = \frac{2e^t \cos t}{-2e^t \sin t} = -\frac{\cos t}{\sin t} = -\frac{1}{\tan t}$$

$\tan t$  and  $-\frac{1}{\tan t}$  are perpendicular (opposite reciprocals), so the angle between  $\mathbf{r}(t)$  and  $\mathbf{a}(t)$  is a right angle  $\rightarrow \boxed{\pi/2}$

$$46. \mathbf{a}(t) = \langle -2, -2 \rangle, \mathbf{v}(1) = \langle 4, 0 \rangle, \text{ and } \mathbf{r}(1) = \langle 3, 3 \rangle$$

$$\mathbf{v}(t) = \langle -2t + C_1, -2t + C_2 \rangle$$

$$-2(1) + C_1 = 4 \quad -2(1) + C_2 = 0$$

$$C_1 = 6$$

$$C_2 = 2$$

$$\mathbf{v}(t) = \langle -2t + 6, -2t + 2 \rangle$$

$$\mathbf{r}(t) = \langle -t^2 + 6t + C_1, -t^2 + 2t + C_2 \rangle$$

$$-t^2 + 6t + C_1 = 3 \quad -t^2 + 2t + C_2 = 3$$

$$-1^2 + 6(1) + C_1 = 3 \quad -1^2 + 2(1) + C_2 = 3$$

$$C_1 = -2$$

$$C_2 = 2$$

$$\boxed{\mathbf{r}(t) = \langle -t^2 + 6t - 2, -t^2 + 2t + 2 \rangle}$$

$$48. x = e^t \cos t, y = e^t \sin t$$

$$a) \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t \cos t + e^t \sin t}{-e^t \sin t + e^t \cos t} \bigg|_{t=\pi} = \frac{e^\pi(-1) + e^\pi(0)}{-e^\pi(0) + e^\pi(-1)} = \frac{-e^\pi}{-e^\pi} = \boxed{1}$$

$$b) \frac{dx}{dt} \bigg|_{t=3} = -e^3 \sin 3 + e^3 \cos 3 \quad \text{speed} = \sqrt{(-e^3 \sin 3 + e^3 \cos 3)^2 + (e^3 \cos 3 + e^3 \sin 3)^2}$$

$$\frac{dy}{dt} \bigg|_{t=3} = e^3 \cos 3 + e^3 \sin 3 \quad = \boxed{28.405}$$

$$c) \int_0^3 \sqrt{(-e^t \sin t + e^t \cos t)^2 + (e^t \cos t + e^t \sin t)^2} dt = \boxed{26.991}$$