Chapter 3 Review: 2–78 e.o.e.

2. $y = 3 - 7x^3 + 3x^7 \to y' = \boxed{-21x^2 + 21x^6}$

6. $s = \dfrac{t^2+1}{1-t^2} \to s' = \dfrac{(1-t^2)2t - (t^2+1)(-2t)}{(1-t^2)^2} = \dfrac{2t - \cancel{2t^3} + \cancel{2t^3} + 2t}{(1-t^2)^2} = \boxed{\dfrac{4t}{(1-t^2)^2}}$

10. $r = \dfrac{\tan\theta}{\theta^3+\theta+1} \to r' = \boxed{\dfrac{(\theta^3+\theta+1)\sec^2\theta - \tan\theta(3\theta^2+1)}{(\theta^3+\theta+1)^2}}$

14. $y = \tan x - \cot x \to y' = \boxed{\sec^2 x + \csc^2 x}$

18. $A = \dfrac{\sqrt{3}}{4}s^2 + \dfrac{3\pi}{8}s^2 \to A' = \dfrac{\sqrt{3}}{2}s + \dfrac{3\pi}{4}s = \boxed{\left(\dfrac{\sqrt{3}}{2} + \dfrac{3\pi}{4}\right)s}$

22. $y = x^{-2}\cos x - 4x^{-3} \to y' = x^{-2}(-\sin x) + \cos x(-2x^{-3}) + 12x^{-4}$

$y' = \boxed{\dfrac{-\sin x}{x^2} - \dfrac{2\cos x}{x^3} + \dfrac{12}{x^4}}$

26. $y = 4x^2(x^{-1} + 3x^{-4}) = 4x + 12x^{-2} \to y' = 4 - 24x^{-3} = \boxed{4 - \dfrac{24}{x^3}}$

30. $y = \dfrac{\sin x \cot x}{\cos x} = \dfrac{\sin x}{\cos x} \cdot \dfrac{\cos x}{\sin x} = 1 \to y' = \boxed{0}$

34. $y = \dfrac{x+5}{2x-7} \to 2x - 7 \neq 0 \to x \neq \frac{7}{2} \to$ Not differentiable at $x = 7/2$ bc there is a vertical asymptote, so the function is discontinuous.

38. $y = \dfrac{x}{x+\sin x} \to y' = \dfrac{x + \sin x - x(1+\cos x)}{(x+\sin x)^2}$

$y'(\pi) = \dfrac{\pi + \cancel{0} - \pi(1+-1)}{(\pi+0)^2} = \dfrac{\pi}{\pi^2} = \boxed{\dfrac{1}{\pi}}$

42. $y = x - x\cos x$

$y' = 1 - x(-\sin x) + \cos x(-1) = 1 + x\sin x - \cos x$

$y'' = x\cos x + \sin x + \sin x = \boxed{x\cos x + 2\sin x}$

46. $y = 4 + \cot x - 2\csc x, \quad x = \pi/2$

$y(\pi/2) = 4 + \frac{0}{1} - 2 \cdot \frac{1}{1} = 4 + 0 - 2 = 2 \rightarrow (\pi/2, 2)$

$y' = -\csc^2 x + 2\csc x \cot x$

$y'(\pi/2) = -1^2 + 2 \cdot 1 \cdot 0 = -1 + 0 = -1 =$ tangent slope $\rightarrow$ normal slope $= 1$

Tangent: $y - 2 = -1(x - \pi/2) \rightarrow \boxed{y = -x + \frac{\pi}{2} + 2}$

Normal: $y - 2 = 1(x - \pi/2) \rightarrow \boxed{y = x - \frac{\pi}{2} + 2}$

50. $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 \rightarrow y' = x^2 - x = 6 \rightarrow x^2 - x - 6 = 0 \rightarrow (x-3)(x+2) = 0$

$x = 3, x = -2 \rightarrow y(3) = \frac{1}{3} \cdot 27 - \frac{1}{2} \cdot 9 = 9 - \frac{9}{2} = \frac{9}{2} \rightarrow \boxed{(3, 9/2)}$

$y(-2) = \frac{1}{3} \cdot -8 - \frac{1}{2} \cdot 4 = \frac{-8}{3} - 2 = \frac{-14}{3} \rightarrow \boxed{(-2, -14/3)}$

54. $f(x) = \begin{cases} 2\sin x, & x \le 0 \\ mx, & x > 0 \end{cases}$

a) Continuous when $2\sin x = mx$ at $x = 0$: $2\sin 0 = m \cdot 0 \rightarrow 0 = 0$, so $\boxed{\text{any } m}$ will make $f(x)$ continuous.

b) Differentiable when $2\cos x = m$ at $x = 0$: $2\cos 0 = m \rightarrow m = 2 \cdot 1 = \boxed{2}$

58. $g(x) = \begin{cases} \frac{x-1}{x}, & -2 \le x < 0 \\ \frac{x+1}{x}, & 0 \le x \le 2 \end{cases}$ Not differentiable at endpoints: $x = -2$, $x = 2$

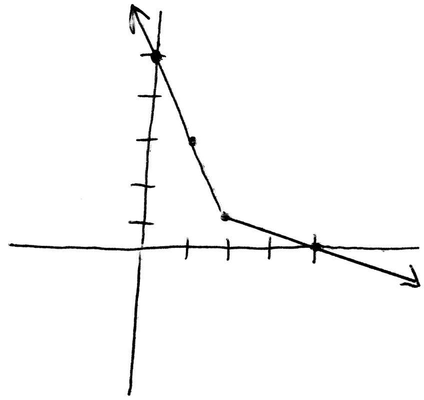
Check $x = 0$: continuity: $\lim_{x \to 0^-} \frac{x-1}{x} = \frac{-1}{0} =$ undefined at $x = 0$

$\lim_{x \to 0^+} \frac{x+1}{x} = \frac{1}{0} =$ undefined at $x = 0$

Since $g(x)$ is not continuous at $x = 0$, then $g(x)$ is not differentiable at $x = 0$.

a) $(-2, 0) \cup (0, 2)$
b) $x = -2, \ x = 2$
c) $x = 0$

62. $f(0)=5, \quad f'(x)=\begin{cases}-2, & x<2\\ -\frac{1}{2}, & x>2\end{cases}$



66. $n=1 \to y=x \to y'=1=1!$

$n=2 \to y=x^2 \to y'=2x \to y''=2=2!$

$n=3 \to y=x^3 \to y'=3x^2 \to y''=6x \to y'''=6=3!$

$n=4 \to y=x^4 \to y'=4x^3 \to y''=12x^2 \to y'''=24x \to y^{4}=24=4!$

n^{th} derivative of $x^n = n(n-1)(n-2)\ldots(2)(1)x^0 = \boxed{n!}$

70. $(f\cdot g)' = f\cdot g' + g\cdot f'$

If $f(0)$ and/or $g(0)$ or negative, then the slope of $f\cdot g$ at $x=0$ could be negative.

74. $p=\left(3-\frac{1}{40}x\right)^2$

a) revenue = fare per rider × number of riders

$r=\left(3-\frac{1}{40}x\right)^2 x = \left(9-\frac{3}{20}x+\frac{1}{1600}x^2\right)x = \boxed{\frac{1}{1600}x^3-\frac{3}{20}x^2+9x}$

b) marginal revenue $= r'(x)$

$\frac{3}{1600}x^2-\frac{3}{10}x+9=0$ at $x=40 \to$ $\boxed{\text{40 people}}$

$p(40)=\left(3-\frac{1}{40}\cdot 40\right)^2=(3-1)^2=2^2=$ $\boxed{\text{\$4 fare}}$

c) The maximum revenue ($r'(x)$ changes from + to −) is obtained when there are 40 passengers. That would be ideal if each bus held 40 passengers instead of 60.

78. $y = \dfrac{4x}{x^2+2}$

$$y' = \frac{(x^2+2)4 - 4x(2x)}{(x^2+2)^2} = \frac{4x^2+8-8x^2}{(x^2+2)^2} = \frac{8-4x^2}{(x^2+2)^2} = 0 \text{ when } 8-4x^2=0$$

$8-4x^2=0 \rightarrow 4x^2=8 \rightarrow x^2=2 \rightarrow x=\pm\sqrt{2}$

$$\left.\begin{array}{l} y(\sqrt{2}) = \dfrac{4\sqrt{2}}{2+2} = \dfrac{\cancel{4}\sqrt{2}}{\cancel{4}} = \sqrt{2} \\ y(-\sqrt{2}) = \dfrac{-4\sqrt{2}}{2+2} = \dfrac{-\cancel{4}\sqrt{2}}{\cancel{4}} = -\sqrt{2} \end{array}\right\rangle \text{Range}: [-\sqrt{2}, \sqrt{2}] \rightarrow \boxed{a=\sqrt{2}}$$