Chapter 3 Review: 1-75 odd

1. $y = x^5 - \frac{1}{8}x^2 + \frac{1}{4}x$

$y' = 5x^4 - \frac{1}{4}x + \frac{1}{4}$

3. $y = 2\sin x \cdot \cos x$

$y' = 2\sin x(-\sin x) + \cos x \cdot 2\cos x = -2\sin^2 x + 2\cos^2 x$

5. $s = (t^2-1)(t^2+1) = t^4 - 1$

$s' = 4t^3$

7. $y = \sqrt{x} + 1 + \frac{1}{\sqrt{x}} = x^{1/2} + 1 + x^{-1/2}$

$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$

9. $r = 5\theta^2 \cdot \sec\theta$

$r' = 5\theta^2 \cdot \sec\theta\tan\theta + \sec\theta \cdot 10\theta$

11. $y = x^2\sin x + x\cos x$

$y' = x^2\cos x + \sin x \cdot 2x + x(-\sin x) + \cos x = (x^2+1)\cos x + x\sin x$

13. $y = \frac{\tan x}{2x^3}$

$y' = \frac{2x^3 \cdot \sec^2 x - \tan x \cdot 6x^2}{(2x^3)^2} = \frac{2x^3 \cdot \sec^2 x - 6x^2 \cdot \tan x}{4x^6} = \frac{x\sec^2 x - 3\tan x}{2x^4}$

15. $y = \frac{1}{\sin x + \cos x}$

$y' = \frac{(\sin x + \cos x)\,0 - 1(\cos x - \sin x)}{(\sin x + \cos x)^2} = \frac{\sin x - \cos x}{(\sin x + \cos x)^2}$

17. $V = \frac{4}{3}\pi r^3 + 8\pi r^2$

$V' = 4\pi r^2 + 16\pi r$

19. $s = \frac{1+\sin t}{1+\tan t}$

$s' = \frac{(1+\tan t)\cos t - (1+\sin t)\sec^2 t}{(1+\tan t)^2}$

21. $s = \frac{t^{-1} + t^{-2}}{t^{-3}} = \frac{t^{-1}}{t^{-3}} + \frac{t^{-2}}{t^{-3}} = t^2 + t$

$s' = 2t + 1$

23. $y = \frac{\sin u}{\csc u} + \frac{\cos u}{\sec u} = \sin u \cdot \sin u + \cos u \cdot \cos u = \sin^2 u + \cos^2 u = 1$

$y' = 0$

25. $y = 2x^{-2}(x^5 - x^3) = 2x^3 - 2x$

$y' = 6x^2 - 2$

27. $y = \frac{1}{\pi^3} t^2 - \pi^2 \cdot t^{-3}$

$y' = \frac{2}{\pi^3} t + \frac{3\pi^2}{t^4}$

29. $y = \sec x \tan x \cos x = \frac{1}{\cancel{\cos x}} \cdot \frac{\sin x}{\cos x} \cdot \cancel{\cos x} = \frac{\sin x}{\cos x} = \tan x$

$y' = \sec^2 x$

31. $y = \frac{\sin x}{x}$

Diff. for $x \neq 0$ (hole at $x = 0 \rightarrow$ point discontinuity)

33. $y = \frac{3\cos x}{x - 2}$

Diff. for $x \neq 2$ (VA at $x = 2 \rightarrow$ infinite discontinuity)

35. $y = \sec x$

$y' = \sec x \cdot \tan x$

$y'(\pi) = \sec \pi \cdot \tan \pi = -1 \cdot 0 = 0$

37. $y = \frac{\cos x}{x}$

$y' = \frac{x(-\sin x) - \cos x}{x^2}$

$y'(\pi) = \frac{\pi(0) - -1}{\pi^2} = \frac{1}{\pi^2}$

39. $y = \frac{1}{\cos x} = \sec x$

$y' = \sec x \cdot \tan x$

$y'' = \sec x \cdot \sec^2 x + \tan x \cdot \sec x \cdot \tan x = \sec^3 x + \sec x \cdot \tan^2 x$

41. $y = x\sin x$

$y' = x\cos x + \sin x$

$y'' = -x\sin x + \cos x + \cos x = 2\cos x - x\sin x$

43. $y = \frac{1}{2}x^4 - \frac{3}{2}x^2 - x$

$y' = 2x^3 - 3x - 1$

$y'' = 6x^2 - 3$

$y''' = 12x$

$y^4 = 12$

$y^5 = 0$

Further derivatives $= 0$

45. $y = 8x^{-2} = \frac{8}{x^2}$ at $x = 2$ $(2, 2)$ Tan: $y - 2 = -2(x-2)$

$y' = -16x^{-3} = \frac{-16}{x^3}$ Normal: $y - 2 = \frac{1}{2}(x-2)$

$y'(2) = \frac{-16}{8} = -2$

47. $y = \sin x + \cos x$ at $x = \pi/4$ $(\pi/4, \sqrt{2})$ Tan: $y = \sqrt{2}$

$y' = \cos x - \sin x$ Normal: $x = \pi/4$

$y'(\pi/4) = \sqrt{2}/2 - \sqrt{2}/2 = 0$

49. $y = 2x^3$ $y(1) = 2 \rightarrow (1, 2)$

$y' = 6x^2 = 6$ $y(-1) = -2 \rightarrow (-1, -2)$

$x^2 = 1$

$x = \pm 1$

51. $y = \frac{6x}{x+1}$

$y' = \frac{(x+1)(6) - 6x(1)}{(x+1)^2} = \frac{6x + 6 - 6x}{(x+1)^2} = \frac{6}{(x+1)^2} = \frac{6}{1}$

$(x+1)^2 = 1$

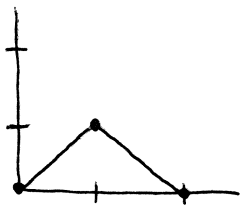
$x + 1 = 1$ $\quad x + 1 = -1$

$x = 0$ $\quad x = -2$

$y(0) = \frac{0}{1} = 0 \rightarrow (0, 0)$

$y(-2) = \frac{-12}{-1} = 12 \rightarrow (-2, 12)$

53. $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$

a) 

b) Yes, $\lim_{x \to 1} f(x) = f(1)$

c) No, corner at $x = 1$

55. $f(x) = x^{4/5}$

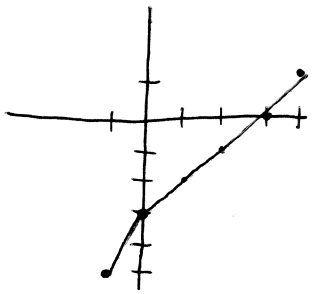
Graph → cusp at $x = 0$

a) $(-\infty, 0) \cup (0, \infty)$

b) $x = 0$

c) None

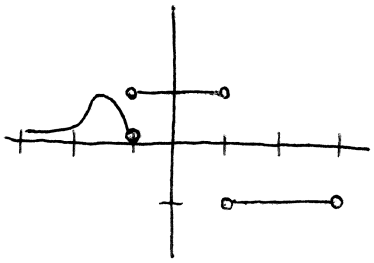
57. $f(x) = \begin{cases} 2x-3, & -1 \le x < 0 \\ x-3, & 0 \le x \le 4 \end{cases}$



a) $(-1, 0) \cup (0, 4)$

b) $x = 0$, $x = -1$, $x = 4$

c) None

59. 

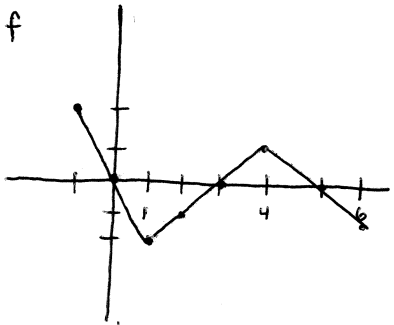
61. a) iii

b) i

c) ii

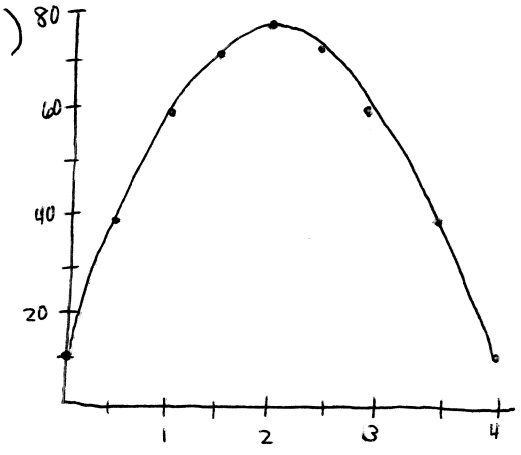
63. $f'(x) = \begin{cases} -2, & x < 1 \\ 1, & 1 < x < 4 \\ -1, & 4 < x < 6 \end{cases}$

$f(-1) = 2$



65.

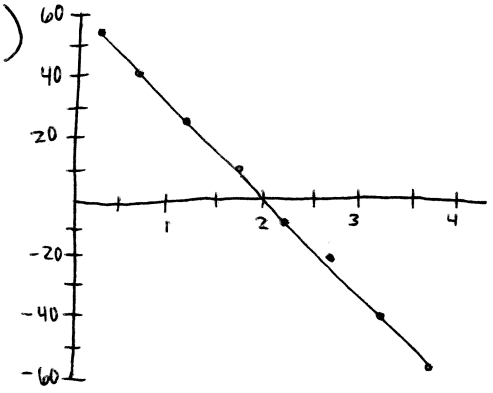
t	0	0.5	1	1.5	2	2.5	3	3.5	4
s	10	38	58	70	74	70	58	38	10

a) 

65. b) Avg. vel $= \frac{\Delta s}{\Delta t}$

t (Mid)	0.25	0.75	1.25	1.75	2.25	2.75	3.25	3.75
v (Avg)	56	40	24	8	-8	-24	-40	-56

c)



d) Computing average velocity using secant lines ($\Delta y/\Delta x$) approximates the true slope of the curve (ds/dt).

67. a) $3f'(x) \to 3f'(1) = 3 \cdot 4 = 12$

b) $xf(x) \to x \cdot f'(x) + f(x) \cdot 1 \to 1 \cdot f'(1) + f(1) = 4 - 3 = 1$

c) $x^2 f(x) \to x^2 \cdot f'(x) + f(x) \cdot 2x \to 1 \cdot f'(1) + f(1) \cdot 2 = 4 + 2(-3) = -2$

d) $\frac{x \cdot f'(x) - f(x) \cdot 1}{x^2} \to \frac{1 \cdot f'(1) - f(1)}{1^2} = \frac{4 - -3}{1} = 7$

e) $\frac{(x^2+2) f'(x) - f(x)(2x)}{(x^2+2)^2} \to \frac{2f'(0) - \cancel{f(0) \cdot 0}}{2^2} = \frac{2 \cdot -2}{4} = -1$

f) $f(x) \cdot f'(x) + f(x) \cdot f'(x) = 2f(0) f'(0) = 2 \cdot 9 \cdot -2 = -36$

69. $f'(0) = +$ and $g'(0) = +$

$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$ at $x = 0 = f'(0) + g'(0) = (+) + (+) = +$

Yes, the sum of 2 positive values is a positive number.

71. $s(t) = 64t - 16t^2$

a) $\frac{ds}{dt} = 64 - 32t$ (velocity)

$\frac{d^2s}{dt^2} = -32$ (acceleration)

b) Max height when vel = 0

$64 - 32t = 0 \to 32t = 64 \to t = 2$ s

c) $v(0) = 64 - 32(0) = 64$ ft/s

d) $s(t) = 64t - 2.6t^2$

$v(t) = 64 - 5.2t = 0$ at $t = 12.308$ s

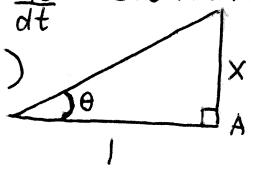
$s(12.308) = 393.846$ ft

73. $V = \pi(10 - \frac{1}{3}x)x^2 = \pi(10x^2 - \frac{1}{3}x^3) = 10\pi x^2 - \frac{1}{3}\pi x^3$

$\frac{dV}{dx} = 20\pi x - \pi x^2 = \pi(20x - x^2)$ in^3 per in

75. $\frac{d\theta}{dt} = -0.6$ rad/s $\quad \frac{dx}{d\theta} = ?$ when $\theta = 0$

a)



$\tan\theta = \frac{x}{1} \rightarrow x = \tan\theta$

$\frac{dx}{d\theta} = \sec^2\theta \quad \frac{d\theta}{dt} = (\sec 0)^2 \cdot \frac{d\theta}{dt} = 1^2 \cdot -0.6 = -0.6$ km/s

b) $\frac{0.6 \text{ rad}}{1 \text{ s}} \frac{1 \text{ rev}}{2\pi \text{ rad}} \frac{60 \text{ s}}{1 \text{ min}} = 5.730$ rev/min