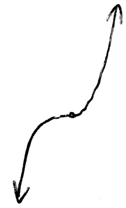
Chapter 5 Review: 2-66 e.o.e.

2. $y = x^3 - 9x^2 - 21x - 11$, $(-\infty, \infty)$

Global (absolute) min/max: $\boxed{\text{DNE}}$



$\lim_{x \to \infty} y = \infty$

$\lim_{x \to -\infty} y = -\infty$

6. $y = e^{x-1} - x$

$y' = e^{x-1} - 1 = 0 \to e^{x-1} = 1 \to x - 1 = 0 \to x = 1$

y' : $- \, - \, 0 \, + + +$ (at $x=1$)

$y'' = e^{x-1} = 0 \to$ never

y'' : $+ + + +$

$y(1) = e^0 - 1 = 1 - 1 = 0 \to (1,0)$

a)	$(1, \infty)$
b)	$(-\infty, 1)$
c)	$(-\infty, \infty)$
d)	None
e)	Local min at $(1,0)$
f)	None

10. $y = \dfrac{x}{x^2 + 2x + 3}$

$y' = \dfrac{(x^2+2x+3)1 - x(2x+2)}{(x^2+2x+3)^2} = \dfrac{x^2 + \cancel{2x} + 3 - 2x^2 - \cancel{2x}}{(x^2+2x+3)^2} = \dfrac{3 - x^2}{(x^2+2x+3)^2} = 0$

$3 - x^2 = 0 \to x^2 = 3 \to x = \pm\sqrt{3}$

y' : $- \, - \, 0 \, + + + \, 0 \, - \, -$ (at $x = -\sqrt{3}$, $x = \sqrt{3}$)

(found in calculator)

$y'' = \dfrac{(x^2+2x+3)^2(-2x) - (3-x^2)2(x^2+2x+3)(2x+2)}{(x^2+2x+3)^4} = 0$ at $x = -2.584$, $x = -0.706$, $x = 3.290$

y'' : $- \, - \, 0 \, + + + + \, 0 \, - \, - \, - \, 0 \, + +$ (at $x = -2.584$, $x = -0.706$, $x = 3.290$)

a)	$(-\sqrt{3}, \sqrt{3})$
b)	$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$
c)	$(-2.584, -0.706) \cup (3.290, \infty)$
d)	$(-\infty, -2.584) \cup (-0.706, 3.290)$
e)	Min at $(-\sqrt{3}, -0.683)$ Max at $(\sqrt{3}, 0.183)$
f)	$(-2.584, -0.573)$ $(-0.706, -0.338)$ $(3.290, 0.161)$

14. $y = -x^5 + \frac{7}{3}x^3 + 5x^2 + 4x + 2$

$y' = -5x^4 + 7x^2 + 10x + 4 = 0$ at $x = -0.578$, $x = 1.692$ (calculator)

y': $- \; - \; 0 \; + \; + \; + \; 0 \; - \; -$
$x = -0.578$ $\quad$ $x = 1.692$

$y'' = -20x^3 + 14x + 10 = 0$ at $x = 1.079$ (calculator)

y'': $+ \; + \; + \; 0 \; - \; - \; -$
$x = 1.079$

a) $(-0.578, 1.692)$
b) $(-\infty, -0.578) \cup (1.692, \infty)$
c) $(-\infty, 1.079)$
d) $(1.079, \infty)$

e) Min at $(-0.578, 0.972)$
Max at $(1.692, 20.517)$

f) $(1.079, 13.606)$

18. $y' = 6(x+1)(x-2) = 0$ at $x = -1$, $x = 2$

y': $+ \; + \; 0 \; - \; - \; 0 \; + \; +$
$x = -1$ $\quad$ $x = 2$

Min at $x = 2$
Max at $x = -1$

$y'' = 6(x+1)\cdot 1 + (x-2)\cdot 6 = 6x + 6 + 6x - 12 = 12x - 6 = 0$ at $x = \frac{1}{2}$

y'': $- \; - \; 0 \; + \; + \; +$
$x = \frac{1}{2}$

POI at $x = \frac{1}{2}$

22. $f'(x) = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$

$f(x) = \boxed{\frac{2}{3}x^{3/2} + 2x^{1/2} + C}$

26. $a = 32$, $v = 20$ and $s = 5$ when $t = 0$

$a(t) = 32$

$v(t) = 32t + C \rightarrow 20 = 0 + C \rightarrow C = 20$

$v(t) = 32t + 20$

$s(t) = 16t^2 + 20t + C$

$5 = 0 + 0 + C \rightarrow C = 5 \rightarrow \boxed{s(t) = 16t^2 + 20t + 5}$

30. $f(x) = e^x + \sin x, \quad a = 0$

$f(0) = e^0 + \sin 0 = 1 + 0 = 1 \rightarrow (0,1)$

$f'(x) = e^x + \cos x \rightarrow f'(0) = e^0 + \cos 0 = 1 + 1 = 2 \rightarrow$ slope

$y - 1 = 2(x - 0) \rightarrow \boxed{y = 2x + 1}$

34. Growth rate increasing: concave up
Growth rate decreasing: concave down
Changes at POI: $\approx$ $\boxed{\text{24 days}}$

38. $s(t) = 3 + 4t - 3t^2 - t^3$

a) $v(t) = \boxed{4 - 6t - 3t^2}$

b) $a(t) = \boxed{-6 - 6t}$

c) At $t = 0$, the particle starts at 3. Using the velocity graph: Velocity is positive until $t = 0.528$, so moving right. After that time, velocity is negative, so moving left.

42. $2\cos x - \sqrt{1+x} = 0$

Newton's Method: 1) Approximate root

2) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ until the 6th decimal place doesn't change

$$x_{n+1} = x_n - \frac{(2\cos x_n - \sqrt{1+x_n})}{\left(-2\sin x_n - \frac{1}{2\sqrt{1+x_n}}\right)}$$

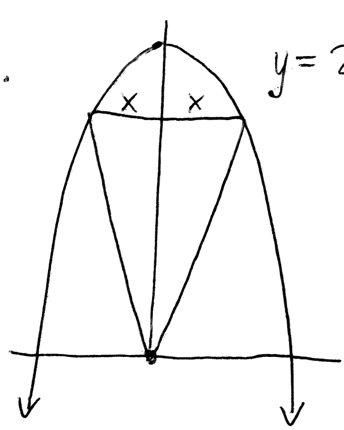
$x_1 = 0.8$

$x_2 = 0.828645$

$x_3 = 0.828361$

$x_4 = \boxed{0.828361}$

46.



$y = 27 - x^2 \qquad A = \frac{1}{2}bh = \cancel{\tfrac{1}{2}} \cdot \cancel{2}x(27 - x^2)$

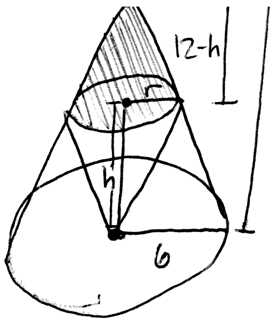
$A = x(27 - x^2) = 27x - x^3$

$A' = 27 - 3x^2 = 0 \rightarrow 3x^2 = 27 \rightarrow x^2 = 9 \rightarrow x = 3$

$A = 3(27 - 3^2) = 3(27 - 9) = 3 \cdot 18$

$A = \boxed{54 \text{ units}^2}$

50.



$V=\frac{1}{3}\pi r^2 h$

Top shaded cone is proportional to whole cone

$\frac{12-h}{r} = \frac{12}{6} \rightarrow 6(12-h) = 12r \rightarrow 72-6h = 12r$

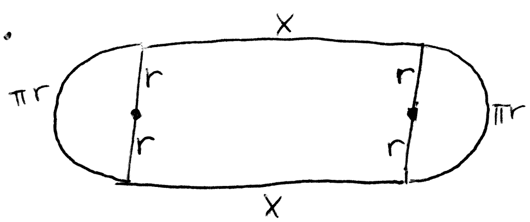
$6h = 72-12r \rightarrow h = 12-2r$

$V = \frac{1}{3}\pi r^2(12-2r) = 4\pi r^2 - \frac{2}{3}\pi r^3$

$V' = 8\pi r - 2\pi r^2 = 2\pi r(4-r) = 0$ at $r=0$ or $\boxed{r=4}$

$h = 12-2r = 12-2(4) \rightarrow 12-8 \rightarrow \boxed{h=4}$

54.



$2x+2\pi r = 400 \rightarrow 2x = 400-2\pi r \rightarrow x = 200-\pi r$

$A = x \cdot 2r \rightarrow A = 2xr$

$A = 2(200-\pi r)r = 400r - 2\pi r^2$

$A' = 400 - 4\pi r = 0 \rightarrow 4\pi r = 400 \rightarrow \pi r = 100 \rightarrow \boxed{r = \frac{100}{\pi}\ m}$

$x = 200-\pi r = 200 - \cancel{\pi} \cdot \frac{100}{\cancel{\pi}} = \boxed{100\ m}$

58. $\frac{dr}{dt} = -\frac{2}{\pi}$ m/s, $\frac{dA}{dt} = ?$, $r = 10$ m

$A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\cancel{\pi} \cdot 10 \cdot \frac{-2}{\cancel{\pi}} = \boxed{-40\ m^2/s}$

62. $\frac{dV}{dt} = -5\ ft^3/min$

a) $\frac{4}{10} = \frac{r}{h} \rightarrow 4h = 10r \rightarrow r = \frac{4h}{10} \rightarrow \boxed{r = \frac{2}{5}h}$

b) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h = \frac{4}{75}\pi h^3$

$\frac{dV}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{dV/dt}{\frac{4}{25}\pi h^2} = \frac{-5}{\left(\frac{4}{25}\pi \cdot 6^2\right)} = -0.276$ ft/min

Water level decreasing at a rate of $\boxed{0.276\ ft/min}$.

66. $S = 6x^2$

a) $dS = 12x\,dx$

Want: $dS \le 0.02S$

$12x\,dx \le 0.02 \cdot 6x^2 \rightarrow 12x\,dx \le 0.12x^2 \rightarrow dx \le \frac{0.12x^{\cancel{2}}}{12\cancel{x}} \rightarrow dx \le 0.01x$

$\boxed{1\%}$ of side length

b) $V = x^3$

$dV = 3x^2\,dx$

$dV \le 3x^2 \cdot 0.01x$

$dV \le 0.03x^3 \rightarrow dV \le 0.03V \rightarrow$ within $\boxed{3\%}$ of actual volume