Chapter 6 Review: 2-56 evens

2. $4x-x^3=0 \rightarrow x(4-x^2)=0 \rightarrow x(2+x)(2-x)=0 \rightarrow x=0,-2,2$

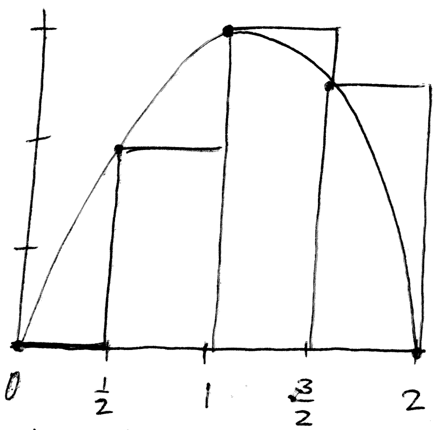
$f(0)=0$

$f(\frac{1}{2}) = 4(\frac{1}{2})-(\frac{1}{2})^3 = 2-\frac{1}{8} = \frac{15}{8}$

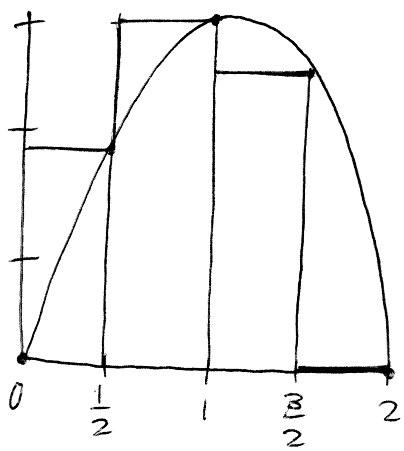
$f(1) = 4-1=3$

$f(\frac{3}{2}) = 4(\frac{3}{2})-(\frac{3}{2})^3 = 6-\frac{27}{8} = \frac{21}{8}$

$f(2) = 8-8 = 0$



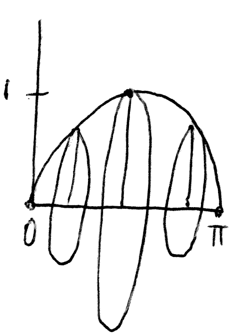
$LRAM = \frac{1}{2}(0+\frac{15}{8}+3+\frac{21}{8}) = \frac{1}{2}(\frac{15}{8}+\frac{24}{8}+\frac{21}{8}) = \frac{1}{2}\cdot\frac{60}{8} = \frac{30}{8} = \boxed{\frac{15}{4}}$

4. 

$RRAM = \frac{1}{2}(\frac{15}{8}+3+\frac{21}{8}+0) = \boxed{\frac{15}{4}}$

6. $\int_0^2 (4x-x^3)dx = (2x^2-\frac{1}{4}x^4)\Big|_0^2 = 8-4 = \boxed{4}$

8. $\int_1^5 \frac{1}{x}dx = \ln x\Big|_1^5 = \ln 5 - \cancel{\ln 1} = \boxed{\ln 5}$

10. 

we can find this volume using disks.

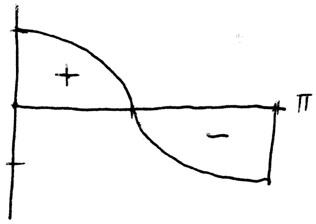
$A = \pi r^2 = \pi(\sin x)^2$

$\pi\int_0^{\pi}(\sin x)^2 dx = \boxed{4.935}$

12. a) $\int_0^{10} x^3\,dx$ b) $\int_0^{10} x\sin x\,dx$ c) $\int_0^{10} x(3x-2)^2\,dx$

d) $\int_0^{10} (1+x^2)^{-1}\,dx$ e) $\int_0^{10} \pi\left(9-\sin^2\left(\frac{\pi}{10}x\right)\right)dx$

14. $y=\cos x,\ [0,\pi] \rightarrow \int_0^{\pi} \cos x\,dx = \sin x\Big|_0^{\pi} = \sin\pi - \sin 0 = 0-0 = 0$ (net area)



Total: $2\int_0^{\pi/2} \cos x\,dx = 2\sin x\Big|_0^{\pi/2} = 2(1-0) = \boxed{2}$ (total area)

16. $\int_2^5 4x\,dx = 2x^2\Big|_2^5 = 50-8 = \boxed{42}$

18. $\int_{-1}^{1} (3x^2-4x+7)\,dx = (x^3-2x^2+7x)\Big|_{-1}^{1} = (1-2+7)-(-1-2-7) = 6+10 = \boxed{16}$

20. $\int_1^2 4x^{-2}\,dx = -4x^{-1}\Big|_1^2 = \frac{-4}{x}\Big|_1^2 = \frac{-4}{2}+\frac{4}{1} = -2+4 = \boxed{2}$

22. $\int_1^4 \frac{1}{t\sqrt{t}}\,dt = \int_1^4 \frac{1}{t^{3/2}}\,dt = \int_1^4 t^{-3/2}\,dt = -2t^{-1/2}\Big|_1^4 = \frac{-2}{\sqrt{t}}\Big|_1^4 = \frac{-2}{2}+\frac{2}{1} = -1+2 = \boxed{1}$

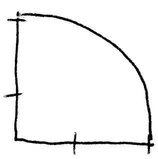
24. $\int_1^e \frac{1}{x}\,dx = \ln x\Big|_1^e = \ln e - \ln 1 = 1-0 = \boxed{1}$

26. $\int_1^2 (x+x^{-2})\,dx = \left(\frac{1}{2}x^2-\frac{1}{x}\right)\Big|_1^2 = \left(2-\frac{1}{2}\right)-\left(\frac{1}{2}-1\right) = \frac{3}{2}+\frac{1}{2} = \boxed{2}$

28. $\int_{-1}^{1} 2x\sin(1-x^2)\,dx = \cos(1-x^2)\Big|_{-1}^{1} = \cos 0 - \cos 0 = 1-1 = \boxed{0}$

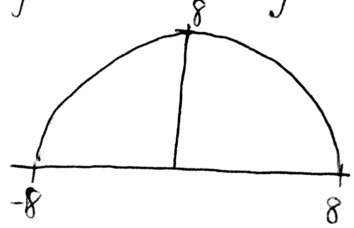
30. $\int_{0}^{2} \sqrt{4-x^2}\,dx$ $\quad y=\sqrt{4-x^2} \to y^2 = 4-x^2 \to x^2+y^2 = 4$

Circle centered at $(0,0)$ with $r=2$



$\frac{1}{4}\pi r^2 = \frac{1}{4}\pi\cdot 2^2 = \boxed{\pi}$

32. $\int_{-8}^{8} 2\sqrt{64-x^2}\,dx = 2\int_{-8}^{8}\sqrt{64-x^2}\,dx$ $\quad y=\sqrt{64-x^2} \to y^2 = 64-x^2 \to x^2+y^2 = 64$



$\cancel{2\cdot\frac{1}{2}}\pi r^2$

$\pi\cdot 8^2 = \boxed{64\pi}$

34. Speed is decreasing, so LRAM = upper and RRAM = lower.

a) Upper = LRAM = $3(5.30+5.25+5.04+\ldots+1.11) = \boxed{103.05 \text{ ft}}$

Lower = RRAM = $3(5.25+5.04+4.71+\ldots+0) = \boxed{87.15 \text{ ft}}$

b) $T = \frac{1}{2}\cdot 3(5.30+2(5.25)+2(5.04)+\ldots+2(1.11)+0) = \boxed{95.1 \text{ ft}}$

36. $\int_{-4}^{0}(x-2)\,dx + \int_{0}^{4} x^2\,dx = \left(\frac{1}{2}x^2-2x\right)\Big|_{-4}^{0} + \left(\frac{1}{3}x^3\right)\Big|_{0}^{4}$

$0-(8+8)+\frac{64}{3}-0 = -16+\frac{64}{3} = \frac{-48}{3}+\frac{64}{3} = \boxed{\frac{16}{3}}$

38. Avg. value $= \frac{\text{Area}}{\Delta x}$

a) $\frac{1}{4}\int_{0}^{4} x^{1/2}\,dx = \frac{1}{4}\cdot\frac{2}{3}x^{3/2}\Big|_{0}^{4} = \frac{1}{6}x^{3/2}\Big|_{0}^{4} = \frac{1}{6}\cdot 4^{3/2} = \frac{1}{6}\cdot 8 = \frac{8}{6} = \boxed{\frac{4}{3}}$

b) $\frac{1}{a}\int_{0}^{a} a x^{1/2}\,dx = \frac{1}{\cancel{a}}\cdot\cancel{a}\cdot\frac{2}{3}x^{3/2}\Big|_{0}^{a} = \frac{2}{3}a^{3/2} - 0 = \boxed{\frac{2}{3}a^{3/2}}$

40. $y=\int_{2}^{7x^2}\sqrt{2+\cos^3 t}\,dt \rightarrow \frac{dy}{dx}=\boxed{\sqrt{2+\cos^3(7x^2)}\cdot 14x}$

42. $y=\int_{x}^{2x}\frac{1}{t^2+1}\,dt \rightarrow \frac{dy}{dx}=\frac{1}{4x^2+1}\cdot 2-\frac{1}{x^2+1}\cdot 1=\boxed{\frac{2}{4x^2+1}-\frac{1}{x^2+1}}$

44. Avg. value of $I(x)=\frac{\text{Area}}{\Delta x}$

$\frac{1}{14}\int_{0}^{14}(600+600t)\,dt=\frac{1}{14}(600t+300t^2)\Big|_{0}^{14}=\frac{1}{14}(600(14)+300(14)^2)=\boxed{4{,}800\text{ cases}}$

4800 cases × 0.04 \$/case $=\boxed{\$\,192/\text{day}}$

46. $g(x)=\int_{0}^{x}f(t)\,dt \rightarrow g'(x)=f(x) \rightarrow g''(x)=f'(x)=+$

a) True b) True c) $g'(1)=f(1)=0 \rightarrow$ True

d) $g''(1)=f'(1)=+ \rightarrow$ concave up $\rightarrow$ not max $\rightarrow$ False

e) $g'(1)=0$ and $g''(1)>0$, so hor. tan. & conc. up $\rightarrow$ min $\rightarrow$ True

f) $g''(1)=f'(1)>0 \rightarrow$ concave up $\rightarrow$ not POI $\rightarrow$ False

g) $g'(1)=f(1)=0 \rightarrow$ True

48. $\frac{dy}{dx}=\frac{\sin x}{x},\ y(5)=3 \rightarrow y(x)=y(5)+\int_{5}^{x}\frac{dy}{dt}\,dt=\boxed{3+\int_{5}^{x}\frac{\sin t}{t}\,dt}$

50. $\frac{dy}{dx}=2x \rightarrow y=x^2+C \rightarrow$ $\boxed{\text{Graph B}}$

52. a) $a=-32 \rightarrow v=-32t+C$ at $t=0,\ v=0 \rightarrow C=0 \rightarrow v=-32t$

$s=-16t^2+C$ at $t=0,\ s=0$ (in plane) $\rightarrow C=0 \rightarrow s=-16t^2$

$s(4)=-16\cdot 4^2=-16\cdot 16=-256 \rightarrow$ A descends 256 ft before opening parachute

6400 ft − 256 ft $=\boxed{6{,}144\text{ ft}}$

52. b) $s(13) = -16 \cdot 13^2 = -2704 \rightarrow$ B descends 2,704 ft before opening parachute

$7000 \text{ ft} - 2704 \text{ ft} = \boxed{4{,}296 \text{ ft}}$

c) A opens parachute after 4 seconds.
B opens parachute after $45 + 13 = 58$ seconds.
Ground = position 0
Position = height when chute opens − rate (time after opening)

A: $6144 - 16(t-4) = 0$ at $t = 388$ s

B: $4296 - 16(t-58) = 0$ at $t = 326.5$ s $\leftarrow$ $\boxed{\text{B lands sooner}}$

54. a) $g(1) = \int_1^1 f(t)\,dt = \boxed{0}$

b) $g(3) = \int_1^3 f(t)\,dt = -\frac{1}{2} \cdot 2 \cdot 1 = \boxed{-1}$

c) $g(-1) = \int_1^{-1} f(t)\,dt = -\int_{-1}^{1} f(t)\,dt = -\frac{1}{4} \cdot \pi \cdot 2^2 = \boxed{-\pi}$

d) $g'(x) = f(x)$
g has max when $g'(x) = f(x) = 0$ or DNE and sign of $g'(x) = f(x)$ changes + to −.
$\boxed{x = 1}$

e) $g(-1) = -\pi \rightarrow (-1, -\pi)$
$g'(-1) = f(-1) = 2$
$y + \pi = 2(x+1)$
$\boxed{y = 2x + 2 - \pi}$

56. $T = \frac{1}{2} \cdot 20(146 + 2(122) + 2(76) + 2(54) + 2(40) + 2(30) + 13) = 8{,}030 \text{ ft}^2$

Volume $= 8030 \cdot 5 = 40{,}150 \text{ ft}^3$

$$\frac{40{,}150 \cancel{\text{ft}^3}}{1} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \boxed{1{,}487.037 \text{ yd}^3}$$