Ch. 6 Review: 35-57 odd, 56

35. The integral finds the area under the curve perfectly using an infinite number of rectangles. The function is the height of the rectangle and dx is the change in x, so width of the rectangle.

37. $0 \le \int_0^1 \sqrt{1+\sin^2 x}\,dx \le \sqrt{2}$

Min y value $= \sqrt{1+(\sin 0)^2} = \sqrt{1+0^2} = \sqrt{1} = 1$

Max y value $= \sqrt{1+(\sin 1)^2} = \sqrt{1+\sin^2 1} = 1.307$

$$\boxed{0 < 1 \le \int_0^1 \sqrt{1+\sin^2 x}\,dx \le 1.307 < \sqrt{2}}$$

Min area $=$ Min $y \times \Delta x = 1 \times (1-0) = 1 \times 1 = 1$

Max area $=$ Max $y \times \Delta x = \sqrt{1+\sin^2 1} \times (1-0) = \sqrt{1+\sin^2 1} \approx 1.307$

39. $y = \int_2^x \sqrt{2+\cos^3 t}\,dt \rightarrow \frac{dy}{dx} = \boxed{\sqrt{2+\cos^3 x}}$

41. $y = \int_x^1 \frac{6}{3+t^4}\,dt = -\int_1^x \frac{6}{3+t^4}\,dt \rightarrow \frac{dy}{dx} = \boxed{\frac{-6}{3+x^4}}$

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$$\int_{25}^{2500} \frac{2}{\sqrt{x}}\,dx = \int_{25}^{2500} 2x^{-1/2}\,dx = 4\sqrt{x}\Big|_{25}^{2500} = 4\sqrt{2500} - 4\sqrt{25} = 4\cdot 50 - 4\cdot 5 = 200 - 20 = 180$$

$\$50 + \$180 = \boxed{\$230}$

45. $\int_0^x (t^3 - 2t + 3)\,dt = 4 \rightarrow \frac{1}{4}t^4 - t^2 + 3t\Big|_0^x = 4 \rightarrow \frac{1}{4}x^4 - x^2 + 3x = 4$

Find intersection on calculator: $\boxed{x = 1.631, -3.091}$

47. $\int_0^1 \sqrt{1+x^4}\,dx = F(x)\Big|_0^1 = \boxed{F(1) - F(0)}$

49. $y = x^2 + \int_1^x \frac{1}{t}\,dt + 1$

i) $y' = 2x + \frac{1}{x} = 2x + x^{-1}$

$y'' = 2 - x^{-2} = \boxed{2 - \frac{1}{x^2}}$

ii) $y(1) = 1^2 + \int_1^1 \frac{1}{t}\,dt + 1 = 1 + 0 + 1 = \boxed{2}$

$y'(1) = 2(1) + \frac{1}{1} = 2 + 1 = \boxed{3}$

51. 5 min $= \frac{1}{12}$ hr bc gal/~~hr~~ $\cdot$ ~~hr~~ = gal

a) $\frac{1}{2} \cdot \frac{1}{12}(2.5 + 2(2.4) + 2(2.3) + 2(2.4) + \ldots + 2(2.4) + 2(2.4) + 2.3) = \boxed{2.417 \text{ gal}}$

b) $\frac{60 \text{ mi}}{2.417 \text{ gal}} = \boxed{24.828 \text{ mpg}}$

53. a) Trapezoid + 2 Rectangle $= h(y_1 + 4y_2 + y_3)$

Trapezoid $= \frac{1}{2} \cdot 2h\,(y_1 + y_3) = h(y_1 + y_3)$

Rectangle $= bh = 2h \cdot y_2$, so twice the rectangle $= 4hy_2$

$T + 2R = h(y_1 + y_3) + 4hy_2 = h(y_1 + y_3 + 4y_2) = \boxed{h(y_1 + 4y_2 + y_3)}$

b) Simpson's Rule is not in our curriculum → skip this part! ☺

55. $\text{NINT}(e^{-x^2/2}, x, -10, 10) = 2.507$
$\text{NINT}(e^{-x^2/2}, x, -1{,}000, 1{,}000) = 2.507$
> same to all decimal places shown

$2.507 = \boxed{\sqrt{2\pi}}$ (guess and check)

56. $V = 5T = 5 \cdot \frac{1}{2} \cdot 20(146 + 2(122) + 2(76) + 2(54) + 2(40) + 2(30) + 13) = 40{,}150 \text{ ft}^3$

$\frac{40{,}150 \cancel{\text{ft}} \cdot \cancel{\text{ft}} \cdot \cancel{\text{ft}}}{1} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \cdot \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} = \boxed{1{,}487.037 \text{ yd}^3}$

57. $V_{RMS} = \sqrt{(V^2)_{AV}}$, $V_{RMS} = \frac{V_{MAX}}{\sqrt{2}}$

a) $\sqrt{(V^2)_{AV}} = V_{RMS} \rightarrow (V^2)_{AV} = (V_{RMS})^2 = \left(\frac{V_{MAX}}{\sqrt{2}}\right)^2 = \boxed{\frac{(V_{MAX})^2}{2}}$

$V_{RMS} = \sqrt{(V^2)_{AV}} = \sqrt{\frac{(V_{MAX})^2}{2}} = \boxed{\frac{V_{MAX}}{\sqrt{2}}}$

b) $V_{RMS} = \frac{V_{MAX}}{\sqrt{2}} \rightarrow V_{MAX} = \sqrt{2} \cdot V_{RMS} = \sqrt{2} \cdot 240 = \boxed{339.411 \text{ volts}}$