Ch. 7 Review: 2-58 e.o.e., 60, 64

2. $\int_1^2 (x + x^{-2})\,dx = \left[\frac{1}{2}x^2 - \frac{1}{x}\right]_1^2 = (2 - 1/2) - (1/2 - 1) = 1.5 + 0.5 = \boxed{2}$

6. $\int_{1/2}^4 \frac{x^2+3x}{x}\,dx = \int_{1/2}^4 (x+3)\,dx = \left[\frac{1}{2}x^2 + 3x\right]_{1/2}^4 = (8+12) - (1/8 + 3/2)$

$\frac{160}{8} - \frac{1}{8} - \frac{12}{8} = \boxed{\frac{147}{8}}$

10. $\int_1^2 \frac{2x+6}{x^2-3x}\,dx \qquad \frac{2x+6}{x^2-3x} = \frac{2x+6}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$

$A(x-3) + B(x) = 2x+6$

$x = 3: \quad 3B = 12 \rightarrow B = 4$

$x = 0: \quad -3A = 6 \rightarrow A = -2$

$\int_1^2 \left(\frac{4}{x-3} - \frac{2}{x}\right) dx = \left[4\ln|x-3| - 2\ln|x|\right]_1^2$

$(4\underbrace{\cancel{\ln 1}}_{0} - 2\ln 2) - (4\ln 2 - 2\underbrace{\cancel{\ln 1}}_{0}) = -2\ln 2 - 4\ln 2 = \boxed{-6\ln 2}$

14. $\int \frac{1}{\theta^2} \sec\frac{1}{\theta} \tan\frac{1}{\theta}\,d\theta \qquad u = \frac{1}{\theta}$

$du = \frac{-1}{\theta^2}\,d\theta$

$d\theta = -\theta^2\,du$

$\int \frac{1}{\cancel{\theta^2}} \sec u \tan u\,(-\cancel{\theta^2})\,du = -\int \sec u \tan u\,du = -\sec u + C = \boxed{-\sec\frac{1}{\theta} + C}$

18. $\int \frac{dt}{t\sqrt{t}} = \int \frac{1}{t^{3/2}}\,dt = \int t^{-3/2}\,dt = -2t^{-1/2} + C = \boxed{\frac{-2}{\sqrt{t}} + C}$

22. $\int x^2 e^{-3x}\,dx$

x^2	e^{-3x}
$2x$	$-\frac{1}{3}e^{-3x}$
2	$\frac{1}{9}e^{-3x}$
0	$-\frac{1}{27}e^{-3x}$

$+$, $-$, $+$

$\boxed{-\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C}$

26. $\frac{dy}{dx} = (x + \frac{1}{x})^2, \; y(1) = 1$

$$\frac{dy}{dx} = x^2 + 2\cdot\cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2}$$

$$y = \frac{1}{3}x^3 + 2x - \frac{1}{x} + C$$

$$1 = \frac{1}{3} + 2 - 1 + C \rightarrow C = -1/3$$

$$\boxed{y = \frac{1}{3}x^3 + 2x - \frac{1}{x} - \frac{1}{3}}$$

30. $r''' = -\cos t, \; r''(0) = r'(0) = r(0) = -1$

$$r'' = -\sin t + C$$

$$-1 = -\sin 0 + C \rightarrow C = -1$$

$$r'' = -\sin t - 1$$

$$r' = \cos t - t + C$$

$$-1 = \cos 0 - \cancel{0} + C \rightarrow C = -2$$

$$r' = \cos t - t - 2$$

$$r = \sin t - \frac{1}{2}t^2 - 2t + C$$

$$-1 = \sin 0 - 0 - 0 + C \rightarrow C = -1$$

$$\boxed{r = \sin t - \frac{1}{2}t^2 - 2t - 1}$$

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34. $\underset{k\;P\;(M\;-\;P)}{\frac{dy}{dx} = 0.001y(100 - y)}$ $\quad k = 0.001, \; M = 100, \; y(0) = 5$

$$P = y = \frac{M}{1 + Ae^{-Mkt}} = \frac{100}{1 + Ae^{-0.1t}}$$

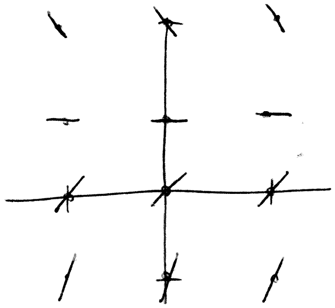
$$5 = \frac{100}{1 + A\cancel{e}^{0}(1)}$$

$$5 + 5A = 100$$

$$5A = 95$$

$$A = 19 \rightarrow \boxed{y = \frac{100}{1 + 19e^{-0.1t}}}$$

38. $\frac{dy}{dx} = 1 - y$



42. $\frac{-xy}{10} = \frac{dy}{dx}$ → slope of 0 when $x = 0$ or $y = 0$

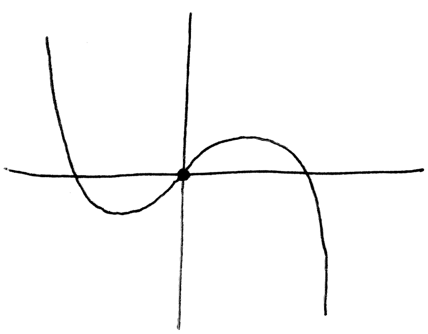
A, ~~B~~, C, ~~D~~

− slope in 1st & 3rd quad; + slope in 2nd & 4th quad → $\boxed{A}$

46. $\int e^{-x^2}dx$

Derivative $= e^{-x^2} > 0$ for all x, so function is always increasing → $\boxed{D}$

50.



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54. carbon-14 half-life = 5700 yr

$y = y_0 e^{-kt}$

$1 = 2e^{-k(5700)}$

$1/2 = e^{-5700k}$

$\ln 1/2 = -5700k \cdot \cancel{\ln e}^{1}$

$k = \frac{\ln 1/2}{-5700} = 0.0001216$

$99.5 = 100e^{-0.0001216t}$

$0.995 = e^{-0.0001216t}$

$\ln 0.995 = -0.0001216t \cdot \cancel{\ln e}^{1}$

$t = \frac{\ln 0.995}{-0.0001216} = \boxed{41.220 \text{ yr}}$

58. $\frac{dy}{dt} = \frac{kA}{V}(c-y)$

a) $\frac{1}{c-y}\,dy = \frac{kA\text{[redacted]}}{V}\,dt$

$-\ln|c-y| = \frac{kA\text{[redacted]}}{V}t + C_1$

$\cancel{\ln}|c-y| = -\frac{kA\text{[redacted]}}{V}t + C_1$

$e \qquad e$

$c-y = C_1 e^{-\frac{kA\text{[redacted]}}{V}t}$

$y = C - C_1 e^{-\frac{kA\text{[redacted]}}{V}t}$

$y_0 = C - C_1 \cancel{e^0}(1) \rightarrow C_1 = C - y_0$

$y = C - (C-y_0)e^{-\frac{kA\text{[redacted]}}{V}t}$

$\boxed{y = C + (y_0 - c)e^{-\frac{kA}{V}t}}$

b) $\lim_{t\to\infty} y(t)$

$\lim_{t\to\infty} C + \frac{y-c}{e^{\frac{kA}{V}t}}$

$C + \frac{y-c}{e^{\infty}} = C + \cancel{\frac{y-c}{\infty}}^{\,0}$

$C + 0 = \boxed{C}$

60. $y'' = 2x\cos(x^2) + 6x, \quad y'(0) = 1, \quad y(0) = 2$

$y' = \sin(x^2) + 3x^2 + C$

$1 = \sin 0 + 0 + C \rightarrow C = 1$

$y' = \sin x^2 + 3x^2 + 1$

$y = \int \sin x^2\,dx + x^3 + x + C$

$2 = 0 + 0 + 0 + C \rightarrow C = 2 \rightarrow \boxed{y = \int \sin x^2\,dx + x^3 + x + 2}$

64. $f(x) = \int_0^x u(t)\,dt, \quad g(x) = \int_3^x (u(t))\,dt$

a) $f'(x) = u(x), \quad g'(x) = u(x)$

$u(x)$ is the derivative of g&f, so g&f are antiderivatives of $u(x)$.

b) $f(x) = g(x) + C \rightarrow C = f(x) - g(x) = \int_0^x u(t)\,dt - \int_3^x u(t)\,dt = \boxed{\int_0^3 u(t)\,dt}$