Ch. 7 Review: 1-33 e.o.o., 37-42 all, 47-50 all, 51-65 odds

1. $\int_0^{\pi/3} \sec^2\theta \, d\theta = \tan\theta \Big|_0^{\pi/3} = \tan\frac{\pi}{3} - \tan 0 = \frac{\sqrt{3}/2}{1/2} - \frac{0}{1} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} - 0 = \boxed{\sqrt{3}}$

5. $\int_0^{\pi/2} 5\sin^{3/2}x \cos x \, dx$

$u = \sin x$
$du = \cos x \, dx$
$dx = \frac{du}{\cos x}$

$\int 5u^{3/2} \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}} = 5\int u^{3/2} du = \cancel{5} \cdot \frac{2u}{\cancel{5}}^{5/2} = 2u^{5/2} = 2(\sin x)^{5/2} \Big|_0^{\pi/2}$

$2(1)^{5/2} - 2(0)^{5/2} = 2\cdot 1 - 2\cdot 0 = 2 - 0 = \boxed{2}$

9. $\int_0^1 \frac{x}{x^2+5x+6} dx$

$\frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$

$A(x+3) + B(x+2) = x$

$x = -3: \quad -B = -3 \rightarrow B = 3$

$x = -2: \quad A = -2$

$\int_0^1 \left(\frac{-2}{x+2} + \frac{3}{x+3}\right) dx = -2\ln|x+2| + 3|x+3| = \ln|x+2|^{-2} + |x+3|^3 = \ln\frac{|x+3|^3}{(x+2)^2}$

$\ln\frac{|x+3|^3}{(x+2)^2} \Big|_0^1 = \ln\left(\frac{4^3}{3^2}\right) - \ln\left(\frac{3^3}{2^2}\right) = \ln\left(\frac{64}{9}\right) - \ln\left(\frac{27}{4}\right) = \ln\left(\frac{64/9}{27/4}\right) = \boxed{\ln\left(\frac{256}{243}\right)}$

13. $\int \frac{t}{t^2+5} dt$

$u = t^2+5$
$du = 2t \, dt$
$dt = \frac{du}{2t}$

$\int \frac{\cancel{t}}{u} \cdot \frac{du}{2\cancel{t}} = \int \frac{1}{2}\cdot\frac{1}{u} du = \frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln|t^2+5| + C = \boxed{\frac{1}{2}\ln(t^2+5) + C}$

7. $\int \frac{1}{x\ln x} dx$

$u = \ln x$
$du = \frac{1}{x} dx$
$dx = x\,du$

$\int \frac{1}{\cancel{x}\cdot u} \cdot \cancel{x}\,du = \int \frac{1}{u} du = \ln|u| + C = \boxed{\ln|\ln x| + C}$

21. $\int e^{3x}\sin x \, dx$

$u = e^{3x}$ $\quad dv = \sin x \, dx$
$du = 3e^{3x}dx$ $\quad v = -\cos x$

$\int e^{3x}\sin x\,dx = -e^{3x}\cos x + \int 3e^{3x}\cos x\,dx$

$u = 3e^{3x}$ $\quad dv = \cos x\,dx$
$du = 9e^{3x}dx$ $\quad v = \sin x$

$\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3e^{3x}\sin x - \int 9e^{3x}\sin x\,dx$

21. (continued)

$$\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3e^{3x}\sin x \cancel{- 9\int e^{3x}\sin x\,dx}$$
$$+9\int e^{3x}\sin x\,dx \qquad\qquad \cancel{+9\int e^{3x}\sin x\,dx}$$

$$10\int e^{3x}\sin x\,dx = -e^{3x}\cos x + 3e^{3x}\sin x$$

$$\int e^{3x}\sin x\,dx = \boxed{\frac{-1}{10}e^{3x}\cos x + \frac{3}{10}e^{3x}\sin x + C}$$

25. $\frac{dy}{dx} = 1 + x + \frac{1}{2}x^2, \quad y(0) = 1$

$y = x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + C$

$1 = 0 + 0 + 0 + C \rightarrow C = 1 \rightarrow \boxed{y = x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + 1}$

29. $y'' = 2x - x^{-2}, \quad y'(1) = 1, \quad y(1) = 0$

$y' = x^2 + x^{-1} + C = x^2 + \frac{1}{x} + C$

$1 = 1^2 + \frac{1}{1} + C \rightarrow C = -1 \rightarrow y' = x^2 + \frac{1}{x} - 1$

$y = \frac{1}{3}x^3 + \ln x - x + C$

$0 = \frac{1}{3} + 0 - 1 + C \rightarrow 0 = \frac{-2}{3} + C \rightarrow C = \frac{2}{3} \rightarrow \boxed{y = \frac{1}{3}x^3 + \ln x - x + \frac{2}{3}}$

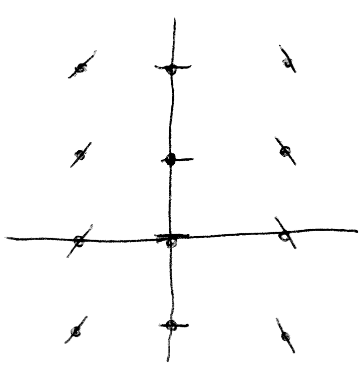
33. $\frac{dy}{dt} = y(1-y), \quad y(0) = 0.1 = \frac{1}{10}$

Like $\frac{dP}{dt} = kP(M-P)$, just with y's instead of P's.

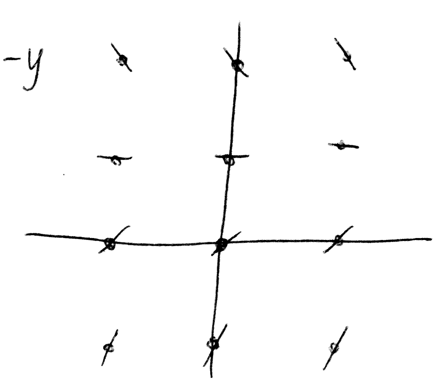
Usually say $P = \frac{M}{1+Ae^{-(Mk)t}}$, so in this case $y = \frac{M}{1+Ae^{-(Mk)t}}$

$\frac{dy}{dt} = \underset{k=1}{1}y(\underset{M=1}{1}-y) \qquad y = \frac{1}{1+Ae^{-t}} \rightarrow \frac{1}{10} = \frac{1}{1+A\cancel{e^0}} \rightarrow A = 9 \rightarrow \boxed{y = \frac{1}{1+9e^{-t}}}$

37. $\frac{dy}{dx} = -x$



38. $\frac{dy}{dx} = 1-y$



39. Graph B
40. Graph D
41. Graph C
42. Graph A
47. Choice iv bc $f(x)=x^2$, so $\frac{dy}{dx}=2x$ and $(1,1)$ is on the graph, so $y(1)=1$.

49. $a=2+6t$, $v(0)=4$

a) $v=2t+3t^2+C$

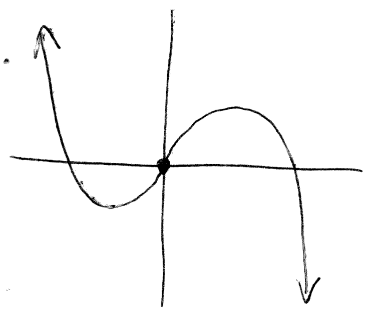
$4=0+0+C \rightarrow C=4 \rightarrow \boxed{v(t)=2t+3t^2+4}$

b) $\int_0^1 (2t+3t^2+4)\,dt = t^2+t^3+4t\Big|_0^1 = (1+1+4)-(0+0+0) = 6-0 = \boxed{6\text{ m}}$

48. $y''=0$, $y'(0)=1$, $y(0)=0$

$y'=C \rightarrow y'=1$

$y=x+C \rightarrow 0=0+C \rightarrow C=0 \rightarrow \boxed{y=x}$

50. 

51. Half-life $= 2.645$ yr

a) $y=y_0e^{kt}$

$1=2e^{k(2.645)}$

$0.5=e^{2.645k}$

$\ln 0.5 = 2.645k \cdot \cancel{\ln e}$

$k=\frac{\ln 0.5}{2.645} = \boxed{-0.262}$ (decay since $k<0$)

b) Mean-life $= \frac{1}{k} = \frac{1}{0.262} = \boxed{3.816\text{ yr}}$

53. $T_0 = 46,\ t_1 = 10,\ T_1 = 39,\ t_2 = 20,\ T_2 = 33,\ T_s = ?$

$$T - T_s = (T_0 - T_s)e^{-kt}$$
$$T - T_s = T_0 e^{-kt} - T_s e^{-kt}$$
$$T_s e^{-kt} - T_s = T_0 e^{-kt} - T$$
$$T_s(e^{-kt} - 1) = T_0 e^{-kt} - T$$
$$T_s = \frac{T_0 e^{-kt} - T}{e^{-kt} - 1}$$

When $t=10, T=39$: $T_s = \dfrac{46e^{-10k} - 39}{e^{-10k} - 1}$

When $t=20, T=33$: $T_s = \dfrac{46e^{-20k} - 33}{e^{-20k} - 1}$

Both equal T_s, so $\dfrac{46e^{-10k} - 39}{e^{-10k} - 1} = \dfrac{46e^{-20k} - 33}{e^{-20k} - 1}$ & find intersection on graph.

$k = 0.01541507$ (x value on graph)

$T_s = \boxed{-3°C}$ (y value on graph)

55. Half-life of carbon-14 = 5700 yr

$$y = y_0 e^{kt}$$
$$1 = 2e^{k \cdot 5700}$$
$$\ln 0.5 = 5700k \cdot \cancel{\ln e}$$
$$k = \frac{\ln 0.5}{5700} = -0.0001216$$

$$10 = 100e^{-0.0001216t}$$
$$0.1 = e^{-0.0001216t}$$
$$\ln 0.1 = -0.0001216t \cdot \cancel{\ln e}$$
$$t = \frac{\ln 0.1}{-0.0001216} = \boxed{18{,}935 \text{ yr}}$$

57. $\dfrac{dL}{dx} = -kL$

$$\int \frac{1}{L}\,dL = \int -k\,dx$$
$$\cancel{e}^{\cancel{\ln}|L|} = e^{-kx + C}$$
$$|L| = e^{-kx} \cdot e^{C}$$
$$L = L_0 e^{-kx}$$

$$1 = 2e^{-k(18)}$$
$$0.5 = e^{-18k}$$
$$\ln 0.5 = -18k \cdot \cancel{\ln e}$$
$$k = \frac{\ln 0.5}{-18} = 0.0385$$

$$1 = 10e^{-0.0385x}$$
$$0.1 = e^{-0.0385x}$$
$$\ln 0.1 = -0.0385x \cdot \cancel{\ln e}$$
$$x = \frac{\ln 0.1}{-0.0385} = \boxed{59.795 \text{ ft}}$$

59. $P(t) = \frac{150}{1+e^{4.3-t}} = \frac{150}{1+e^{4.3}\cdot e^{-t}} = \frac{150}{1+73.700e^{-t}} = \frac{M}{1+Ae^{-(Mk)t}}$

a) $M = \boxed{150}$ students

$Mk = 1 \rightarrow k = \frac{1}{M} = \boxed{\frac{1}{150}}$

b) $P(0) = \frac{150}{1+e^{4.3}} = 2.008 \approx \boxed{2 \text{ students}}$ intially infected with the flu

c) $125 = \frac{150}{1+e^{4.3-t}} \rightarrow 1+e^{4.3-t} = \frac{150}{125} \rightarrow 1+e^{4.3-t} = \frac{6}{5} \rightarrow e^{4.3-t} = \frac{1}{5}$

$(4.3-t)\cdot \cancel{\ln e} = \ln(1/5) \rightarrow 4.3-t = \ln 1/5 \rightarrow t = 4.3 - \ln(\frac{1}{5}) = \boxed{5.909 \text{ days}}$

61. $\frac{dP}{dt} = 0.002P(1-\frac{P}{800}) = 0.002P(\frac{800}{800}-\frac{P}{800}) = \left(\frac{0.002}{800}\right)P(800-P) = kP(M-P$

$\underset{\uparrow}{\frac{0.002}{800}} = k$, $\underset{\uparrow}{800} = M$

$P(0) = 50$

$P = \frac{M}{1+Ae^{-(Mk)t}} \rightarrow 50 = \frac{800}{1+A\cancel{e}^0} \rightarrow 1+A = \frac{800}{50} \rightarrow 1+A = 16 \rightarrow A = 15$

$$\boxed{P = \frac{800}{1+15e^{-0.002t}}}$$

63. a) $A = A_0(1+\frac{r}{n})^{nt}$, $n=1$ for annually

$20{,}000 = 10{,}000(1+\frac{0.063}{1})^{1t}$

$2 = 1.063^t \rightarrow \ln 2 = t\cdot \ln 1.063 \rightarrow t = \frac{\ln 2}{\ln 1.063} = \boxed{11.345 \text{ yr}}$

b) $A = A_0 e^{rt}$

$20{,}000 = 10{,}000e^{0.063t}$

$2 = e^{0.063t} \rightarrow \ln 2 = 0.063t\cdot \cancel{\ln e} \rightarrow t = \frac{\ln 2}{0.063} = \boxed{11.002 \text{ yr}}$

65. a) $\boxed{P = \frac{272{,}286.352}{1+302.687e^{-0.210t}}}$

b) $\lim_{t\to\infty} P = \frac{272{,}286.352}{1+302.687(\frac{1}{\infty})} = \frac{272{,}286.352}{1+0} = \boxed{272{,}286 \text{ people}}$

c) $\frac{dP}{dt} = kP(M-P) = \boxed{7.712\times10^{-7}P(272{,}286.352-P)}$

$M = 272{,}286.352$

$M\cdot k = 0.210 \rightarrow k = \frac{0.210}{M} = 7.712\times10^{-7}$

65. d) The regression equation is less reliable the fewer data points that were used to create it. With the population of 260,283 in 2000 (50, 260,283), the carrying capacity is now 267,313 people. This estimate still isn't very accurate because the population was actually 270,951 in 2003.