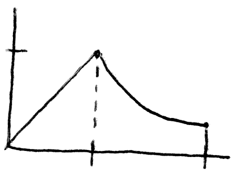
Ch. 8 Review: 2-22 e.o.e., 28, 30, 39, 50

2. $\displaystyle\int_0^7 (4+0.001t^4)\,dt = \left[4t + 0.0002t^5\right]_0^7 \approx \boxed{31.361 \text{ gal}}$

6. 

$y = x$
$y = 1/x^2$
$y = 0$
$x = 2$

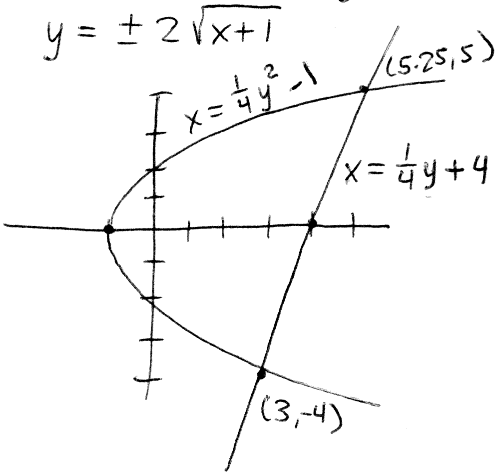
$$\int_0^1 x\,dx + \int_1^2 \frac{1}{x^2}\,dx = \left[\tfrac{1}{2}x^2\right]_0^1 + \left[\tfrac{-1}{x}\right]_1^2$$

$$\tfrac{1}{2} + \left(\tfrac{-1}{2} + 1\right) = \tfrac{1}{2} + \tfrac{1}{2} = \boxed{1}$$

10. $4x = y^2 - 4$, $4x = y + 16$

$y^2 = 4x + 4$ $\qquad y = 4x - 16$

$y = \pm 2\sqrt{x+1}$



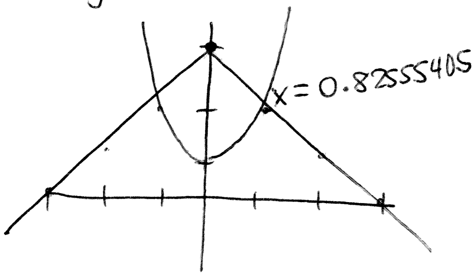
Right - Left

$$\int_{-4}^{5} \left(\tfrac{1}{4}y + 4 - \tfrac{1}{4}y^2 + 1\right)dy = \int_{-4}^{5} \left(\tfrac{1}{4}y - \tfrac{1}{4}y^2 + 5\right)dy$$

$$\left[\tfrac{1}{8}y^2 - \tfrac{1}{12}y^3 + 5y\right]_{-4}^{5}$$

$$\left(\tfrac{25}{8} - \tfrac{125}{12} + 25\right) - \left(2 + \tfrac{64}{12} - 20\right) = \boxed{30.375}$$

14. $y = \sec^2 x$, $y = 3 - |x| = \begin{cases} 3+x, & x \le 0 \\ 3-x, & x > 0 \end{cases}$



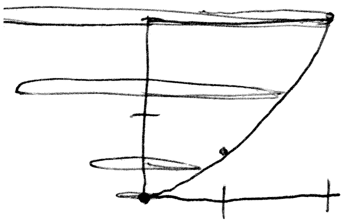
Total = 2× Right side

$$2\int_0^{0.826} (3 - x - \sec^2 x)\,dx = 2\left[3x - \tfrac{1}{2}x^2 - \tan x\right]_0^{0.826} = \boxed{2.104}$$

18. Total = 2× Right side

$$2\int_0^{1.8932983} \left(3^{1-x^2} - \tfrac{1}{10}x^2 + \tfrac{3}{10}\right)dx = \boxed{5.731}$$

22. $y = \frac{1}{2}x^2$ $\quad x^2 = 2y$ $\quad x = \sqrt{2y}$



a) $\pi \int_0^2 \sqrt{2y}^2\, dy = \pi \int_0^2 2y\, dy = \pi \left[y^2\right]_0^2 = \boxed{4\pi}$

b) $\left[\pi y^2\right]_0^k = \pi k^2 - 0 = \boxed{\pi k^2}$

c) $V = \pi k^2$

$\frac{dV}{dt} = 2\pi k \frac{dk}{dt} \rightarrow \frac{dk}{dt} = \frac{dV/dt}{2\pi k} = \frac{\cancel{2}}{\cancel{2}\pi(1)} = \boxed{\frac{1}{\pi} \text{ units/sec}}$

28. $y = x^3 - x, \; y = x - x^3$

Length of $x^3 - x$ from $[-1, 1]$ + Length of $x - x^3$ from $[-1, 1]$

$L = \int_a^b \sqrt{1 + (dy/dx)^2}\, dx$

$\int_{-1}^{1} \sqrt{1 + (3x^2 - 1)^2}\, dx + \int_{-1}^{1} \sqrt{1 + (1 - 3x^2)^2}\, dx = \boxed{5.245}$

30. a) $y = k\sin x$ and $y = \sin kx$ on $[0, 2\pi]$

$\int_0^{2\pi} \sqrt{1 + (k\cos x)^2}\, dx$ vs. $\int_0^{2\pi} \sqrt{1 + (k\cos kx)^2}\, dx$

b) $y = k\sin x$ and $y = \sin x$ on $[0, 2\pi]$

$\int_0^{2\pi} \sqrt{1 + (k\cos x)^2}\, dx$ vs. $\int_0^{2\pi} \sqrt{1 + (\cos x)^2}\, dx$

$1 + k^2\cos^2 x = 1 + \cos^2 x$ only if $k = 1$, but not true for all $k > 0$.

Since b is not true for all $k > 0$, then a must be true.

39. $\sqrt{x} + \sqrt{y} = \sqrt{6}$

$\sqrt{y} = \sqrt{6} - \sqrt{x}$

$y = (\sqrt{6} - \sqrt{x})^2$ = side length of square

Area of $\square = s^2 = (\sqrt{6} - \sqrt{x})^4$

$$\int_0^6 (\sqrt{6} - \sqrt{x})^4 dx = \boxed{14.400}$$

50. $(1,1)$, $L = \ln x + f(x) - 1$

$$\int_1^x \sqrt{1 + (dy/dx)^2}\, dx = \ln x + f(x) - 1$$

$$\sqrt{1 + (dy/dx)^2} = \frac{1}{x} + f'(x)$$

$$1 + (f'(x))^2 = \left(\frac{1}{x} + f'(x)\right)^2$$

$$1 + \cancel{(f'(x))^2} = \frac{1}{x^2} + \frac{2}{x} f'(x) + \cancel{(f'(x))^2}$$

$$1 = \frac{1}{x^2} + \frac{2}{x} f'(x)$$

$$\frac{2}{x} f'(x) = 1 - \frac{1}{x^2}$$

$$2f'(x) = x - \frac{1}{x}$$

$$f'(x) = \frac{1}{2}x - \frac{1}{2x}$$

$$f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x + C$$

$$1 = \frac{1}{4} - 0 + C \rightarrow C = 3/4$$

$$\boxed{f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x + \frac{3}{4}}$$