Chapter 9 Review: 2-50 e.o.e., 52

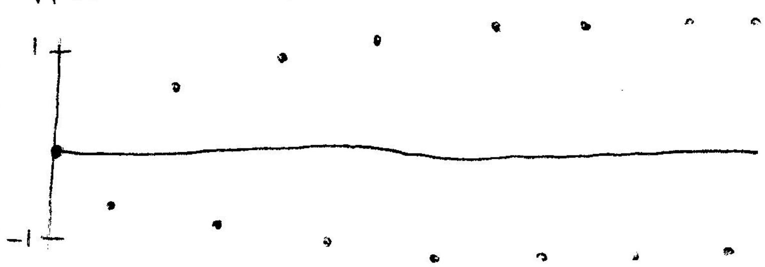
2. $a_1 = -3, \quad a_n = 2a_{n-1} \rightarrow$ Geometric w/ $r = 2$

$$\boxed{-3, -6, -12, -24, \ldots -3(2)^{n-1}}$$

6. $a_n = (-1)^{n-1} \cdot \dfrac{n-1}{n}$

$a_1 = 0, \quad a_2 = -\frac{1}{2}, \quad a_3 = \frac{2}{3}, \quad a_4 = \frac{-3}{4}, \quad a_5 = \frac{4}{5}, \quad a_6 = \frac{-5}{6}$

$$\lim_{n \to \infty} (-1)^{n-1} \cdot \frac{n-1}{n} = \pm 1$$



10. $$\lim_{t \to 0} \frac{\tan 3t}{\tan 5t} = \frac{0}{0} \rightarrow \lim_{t \to 0} \frac{3\sec^2 3t}{5\sec^2 5t} = \frac{3 \cdot 1}{5 \cdot 1} = \boxed{\frac{3}{5}}$$

14. $$\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^x = 1^\infty \rightarrow \text{use } \ln$$

$$x \cdot \ln\left(1 + \frac{3}{x}\right) = \frac{\ln\left(1 + \frac{3}{x}\right)}{\frac{1}{x}} \rightarrow \lim_{x \to \infty} \frac{\ln\left(1 + \frac{3}{x}\right)}{\frac{1}{x}} = \frac{\ln 1}{\frac{1}{\infty}} = \frac{0}{0} = ?$$

$$\lim_{x \to \infty} \frac{\frac{1}{1 + 3/x} \cdot \frac{\cancel{-}3}{\cancel{x^2}}}{\frac{\cancel{-1}}{\cancel{x^2}}} = \lim_{x \to \infty} \frac{3}{1 + \frac{3}{x}} = \frac{3}{1 + \cancel{\frac{3}{\infty}}} = \frac{3}{1} = 3$$

$$\cancel{\ln} \lim_{x \to \infty} f(x) = 3 \rightarrow \lim_{x \to \infty} f(x) = \boxed{e^3}$$
$\cancel{e}$ $\qquad e$

18. $$\lim_{x \to 0^+} \left(1 + \frac{1}{x}\right)^x = \left(1 + \frac{1}{0}\right)^0 = (1 + \infty)^0 = \infty^0 \rightarrow \text{use } \ln$$

$$x \cdot \ln\left(1 + \frac{1}{x}\right) \rightarrow \lim_{x \to 0^+} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = \frac{\ln \infty}{\infty} = \frac{\infty}{\infty} = ?$$

$$\lim_{x \to 0^+} \frac{\frac{1}{1 + 1/x} \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \lim_{x \to 0^+} \frac{1}{1 + \frac{1}{x}} = \frac{1}{1 + \frac{1}{0}} = \frac{1}{1 + \infty} = \frac{1}{\infty} = 0$$

$$\cancel{\ln} \lim_{x \to 0^+} f(x) = 0 \rightarrow \lim_{x \to 0^+} f(x) = e^0 = \boxed{1}$$
$\cancel{e}$ $\qquad e$

22. $\lim\limits_{x\to\infty} \dfrac{3x^2 - x + 1}{x^4 - x^3 + 2} = \lim\limits_{x\to\infty} \dfrac{3x^2}{x^4} = \lim\limits_{x\to\infty} \dfrac{3}{x^2} = \dfrac{3}{\infty^2} = \boxed{0}$

26. $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{100}\cancel{x}}{\frac{\cancel{x}}{e^x}} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{100}}{\frac{1}{e^x}} = \lim\limits_{x\to\infty} \dfrac{1}{100} \cdot \dfrac{e^x}{1} = \dfrac{e^\infty}{100} = \dfrac{\infty}{100} = \infty$

Therefore, $f(x)$ grows $\boxed{\text{faster}}$ than $g(x)$.

30. $f(x) = 3^{-x}$, $g(x) = 2^{-x}$

$\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to\infty} \dfrac{1/3^x}{1/2^x} = \lim\limits_{x\to\infty} \dfrac{1}{3^x} \cdot \dfrac{2^x}{1} = \lim\limits_{x\to\infty} \dfrac{2^x}{3^x} = \lim\limits_{x\to\infty} \left(\dfrac{2}{3}\right)^x = \left(\dfrac{2}{3}\right)^\infty = 0$

Therefore, $f(x)$ grows $\boxed{\text{slower}}$ than $g(x)$.

34. $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$, $g(x) = \frac{1}{x^2}$

$\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to\infty} \dfrac{\sin^{-1}\left(\frac{1}{x}\right)}{\frac{1}{x^2}} = \dfrac{\sin^{-1} 0}{0} = \dfrac{0}{0} = ?$

$\lim\limits_{x\to\infty} \dfrac{\frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot \frac{-1}{x^2}}{\frac{-2}{x^3}} = \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{1-\frac{1}{x^2}}} \cdot \dfrac{\cancel{-1}}{\cancel{x^2}} \cdot \dfrac{\cancel{-}x^{\cancel{3}}}{2} = \lim\limits_{x\to\infty} \dfrac{x}{2\sqrt{1-\frac{1}{x^2}}}$

$\dfrac{\infty}{2\sqrt{1-\cancel{\frac{1}{\infty^2}}}} = \dfrac{\infty}{2\sqrt{1}} = \dfrac{\infty}{2} = \infty$

Therefore, $f(x)$ grows $\boxed{\text{faster}}$ than $g(x)$.

38. $\displaystyle\int_1^\infty \frac{1}{x^2+7x+12}\,dx \qquad \frac{1}{x^2+7x+12} = \frac{1}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$

$A(x+4) + B(x+3) = 1 \rightarrow \quad x=-4: \ -B = 1 \rightarrow B = -1$

$x = -3: \ A = 1$

$\displaystyle\int_1^\infty \left(\frac{1}{x+3} - \frac{1}{x+4}\right)dx = \ln|x+3| - \ln|x+4| = \ln\left|\frac{x+3}{x+4}\right| \Bigg|_1^\infty$

$\ln\left|\dfrac{\infty + \cancel{3}}{\infty + \cancel{4}}\right| - \ln\left(\dfrac{4}{5}\right) = \ln 1 - \ln\dfrac{4}{5} = 0 - \ln\dfrac{4}{5} = -\ln\dfrac{4}{5} = \ln\left(\dfrac{4}{5}\right)^{-1} = \boxed{\ln\left(\dfrac{5}{4}\right)}$

42. $\displaystyle\int_{-1}^{1} \frac{1}{y^{2/3}}\,dy = \int_{-1}^{1} y^{-2/3}\,dy = 3y^{1/3}\Big|_{-1}^{1} = 3\sqrt[3]{y}\Big|_{-1}^{1} = 3\sqrt[3]{1} - 3\sqrt[3]{-1} = 3+3 = \boxed{6}$

46. $\displaystyle\int_{-\infty}^{0} xe^{3x}\,dx$

$$\begin{array}{c|c} x & e^{3x} \\ 1 & +\ \tfrac{1}{3}e^{3x} \\ 0 & -\ \tfrac{1}{9}e^{3x} \end{array}$$

$\displaystyle \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\Big|_{-\infty}^{0}$

$\displaystyle \left(0 - \frac{1}{9}\right) - \left(\cancel{\frac{-\infty}{3e^{3\infty}}} - \cancel{\frac{1}{9e^{3\infty}}}\right) = \frac{-1}{9} - 0 + 0 = \boxed{\frac{-1}{9}}$

50. $\displaystyle\int_{1}^{\infty} \frac{1}{e^t\sqrt{t}}\,dt \le \int_{1}^{\infty} \frac{1}{e^t}\,dt$

$\displaystyle\int_{1}^{\infty} \frac{1}{e^t}\,dt = \int_{1}^{\infty} e^{-t}\,dt = -e^{-t}\Big|_{1}^{\infty} = \frac{-1}{e^t}\Big|_{1}^{\infty} = \frac{-1}{\cancel{e^{\infty}}} + \frac{1}{e^1} = 0 + \frac{1}{e} = \frac{1}{e} \rightarrow$ Converges

$\displaystyle\int_{1}^{\infty} \frac{1}{e^t\sqrt{t}}$ also $\boxed{\text{converges}}$ by the Direct Comparison Test.

52. $\underline{13}$, 11.5, $\underline{10}$, $\underline{8.5}$, $\underline{7}$, 5.5

$11.5 - 5.5 = 6$

$\frac{6}{4} = 1.5$

$a_n = a_1 + (n-1)d$

$a_n = 13 + (n-1)(-1.5) = 13 - 1.5n + 1.5 = 14.5 - 1.5n$

$$\boxed{\begin{array}{l} a)\ 13 = a_1 \\ b)\ -1.5 = d \\ c)\ a_n = 14.5 - 1.5n \end{array}}$$