Ch. 9 Review : 1–53 e.o.o.

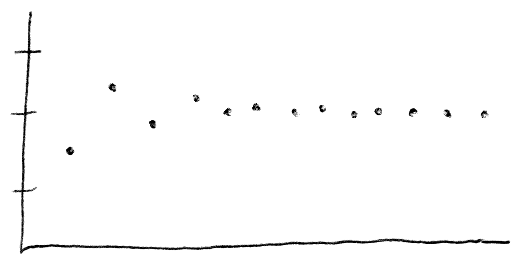
1. $a_n = (-1)^n \dfrac{n+1}{n+3}$

$a_1 = (-1)^1 \cdot \dfrac{1+1}{1+3} = -1 \cdot \dfrac{2}{4} = \boxed{\dfrac{-1}{2}}$ $\qquad a_2 = (-1)^2 \cdot \dfrac{2+1}{2+3} = 1 \cdot \dfrac{3}{5} = \boxed{\dfrac{3}{5}}$

$a_3 = (-1)^3 \cdot \dfrac{3+1}{3+3} = -1 \cdot \dfrac{4}{6} = \boxed{\dfrac{-2}{3}}$ $\qquad a_4 = (-1)^4 \cdot \dfrac{4+1}{4+3} = 1 \cdot \dfrac{5}{7} = \boxed{\dfrac{5}{7}}$

$a_{40} = (-1)^{40} \cdot \dfrac{40+1}{40+3} = 1 \cdot \dfrac{41}{43} = \boxed{\dfrac{41}{43}}$

5. $a_n = \dfrac{2^{n+1} + (-1)^n}{2^n}$



9. $\lim\limits_{t \to 0} \dfrac{t - \ln(1+2t)}{t^2} = \dfrac{0-0}{0} \to \lim\limits_{t \to 0} \dfrac{1 - \dfrac{2}{1+2t}}{2t} = \dfrac{1 - \dfrac{2}{1}}{0} = \dfrac{-1}{0} = \boxed{\infty}$ (diverges

13. $\lim\limits_{x \to \infty} x^{1/x} = \infty^{1/\infty} = \infty^0$

$\dfrac{1}{x} \cdot \ln x = \dfrac{\ln x}{x}$

$\lim\limits_{x \to \infty} \dfrac{\ln x}{x} = \dfrac{\infty}{\infty} \to \lim\limits_{x \to \infty} \dfrac{1/x}{1} = \lim\limits_{x \to \infty} \dfrac{1}{x} = \dfrac{1}{\infty} = 0$

$\cancel{\ln} \lim\limits_{x \to \infty} x^{1/x} = \underset{e}{0} \to \lim\limits_{x \to \infty} x^{1/x} = e^0 = \boxed{1}$

17. $\lim\limits_{x \to 1} \left(\dfrac{1}{x-1} - \dfrac{1}{\ln x} \right) = \dfrac{1}{0} - \dfrac{1}{0} = \infty - \infty$

$\lim\limits_{x \to 1} \dfrac{\ln x - (x-1)}{\ln x (x-1)} = \lim\limits_{x \to 1} \dfrac{\ln x - x + 1}{\ln x (x-1)} = \dfrac{0 - 1 + 1}{0(0)} = \dfrac{0}{0}$

$\lim\limits_{x \to 1} \dfrac{\frac{1}{x} - 1}{\ln x + \frac{(x-1)}{x}} = \dfrac{1-1}{0 + \frac{0}{1}} = \dfrac{0}{0}$

17. (continued)
Multiply every part by x.

$$\lim_{x\to 1} \frac{1-x}{x\ln x + x - 1} = \frac{1-1}{1(0)+1-1} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{-1}{\cancel{x}\cdot\frac{1}{\cancel{x}} + \ln x + 1} = \frac{-1}{1+0+1} = \boxed{\frac{-1}{2}}$$

21. $$\lim_{x\to\infty} \frac{x^3-3x^2+1}{2x^2+x-3} = \lim_{x\to\infty} \frac{x^3}{2x^2} = \lim_{x\to\infty} \frac{x}{2} = \frac{\infty}{2} = \boxed{\infty}$$

25. $$\lim_{x\to\infty} \frac{x+\frac{1}{x}}{x} \to \lim_{x\to\infty} \frac{1+\frac{-1}{x^2}}{\cancel{1}} = 1 - \frac{1}{\infty^2} = 1 - 0 = \boxed{1} \to \boxed{\text{same rate}}$$

29. $f(x) = x^{\ln x}$, $g(x) = x^{\log_2 x} = x^{(\ln x/\ln 2)}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{x^{\ln x}}{x^{\frac{\ln x}{\ln 2}}} = \lim_{x\to\infty} x^{\ln x - \frac{\ln x}{\ln 2}} = \lim_{x\to\infty} x^{\ln x\left(1-\frac{1}{\ln 2}\right)}$$

$$\lim_{x\to\infty} x^{\ln x(-1)\left(\frac{1}{\ln 2}-1\right)} = \lim_{x\to\infty} (x^{-1})^{\ln x\left(\frac{1}{\ln 2}-1\right)} = \lim_{x\to\infty} \left(\frac{1}{x}\right)^{\ln x\left(\frac{1}{\ln 2}-1\right)}$$

$$\left(\frac{1}{\infty}\right)^{\ln\infty\left(\frac{1}{\ln 2}-1\right)} = 0^{\infty} = \boxed{0} \to \boxed{\begin{array}{l} x^{\ln x}: \text{slower} \\ x^{\log_2 x}: \text{faster} \end{array}}$$

33. $f(x) = \tan^{-1}(1/x)$, $g(x) = 1/x$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\tan^{-1}(1/x)}{1/x} = \frac{\tan^{-1} 0}{1/\infty} = \frac{0}{0}$$

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x^2}} \cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \lim_{x\to\infty} \frac{1}{1+\frac{1}{x^2}} = \frac{1}{1+\frac{1}{\infty^2}} = \frac{1}{1+0} = \boxed{1} \to \boxed{\text{same rate}}$$

37. $\displaystyle\int_1^\infty x^{-3/2}dx = \lim_{b\to\infty}\int_1^b x^{-3/2}dx = \lim_{b\to\infty} -2x^{-1/2}\Big|_1^b = \lim_{b\to\infty} \frac{-2}{\sqrt{x}}\Big|_1^b$

$\displaystyle\lim_{b\to\infty}\left(\frac{-2}{\sqrt{b}} + \frac{2}{\sqrt{1}}\right) = \frac{-2}{\sqrt{\infty}} + \frac{2}{1} = 0+2 = \boxed{2}$

41. $\displaystyle\int_0^1 \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv}$ is improper bc $\ln x$ has a vertical asymptote at $x=0$.

$u = \ln x \qquad dv = 1dx$

$du = \frac{1}{x}dx \qquad v = x$

$\displaystyle uv - \int v\,du = x\ln x - \int \cancel{x}\cdot\frac{1}{\cancel{x}}dx = x\ln x - \int 1dx = x\ln x - x$

$\displaystyle\lim_{a\to 0^+}\int_a^1 \ln x\,dx = \lim_{a\to 0^+}(x\ln x - x)\Big|_a^1 = \lim_{a\to 0^+}\left((1\overset{0}{\cancel{\ln 1}}-1)-(a\ln a - a)\right)$

$-1 - 0\ln 0 + 0 = -1 - 0 + 0 = \boxed{-1}$

45. $\displaystyle\int_0^\infty x^2e^{-x}dx$

x^2	$+$	e^{-x}
$2x$	$-$	$-e^{-x}$
2	$+$	e^{-x}
0		$-e^{-x}$

$-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$

$e^{-x}(-x^2-2x-2)$

$-e^{-x}(x^2+2x+2)$

$\displaystyle\lim_{b\to\infty}\int_0^b x^2e^{-x}dx = \lim_{b\to\infty} -e^{-x}(x^2+2x+2)\Big|_0^b$

$\displaystyle\lim_{b\to\infty}\left(-e^{-b}(b^2+2b+2) + e^0(\cancel{0}+\cancel{0}+2)\right) = -e^{-\infty}(\infty^2+2\infty+2)+1(2)$

$\displaystyle\frac{\infty^2+2\infty+2}{-e^{\infty}} + 2 = \frac{\text{slower}}{\text{faster}} + 2 = 0+2 = \boxed{2}$

49. $\displaystyle\int_1^\infty \frac{\ln z}{z}\,dz = \int_1^\infty \frac{\ln x}{x}\,dx = \int_1^e \frac{\ln x}{x}\,dx + \int_e^\infty \frac{\ln x}{x}\,dx$

$$\int_1^e \frac{\ln x}{x}\,dx + \int_e^\infty \frac{\ln x}{x}\,dx > \int_1^e \frac{\ln x}{x}\,dx + \int_e^\infty \frac{1}{x}\,dx$$

bc $\ln e = 1$, and $\ln x > 1$ when $x > e$

$$\lim_{b\to\infty} \int_e^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln x \Big|_e^b = \lim_{b\to\infty} (\ln b - \ln e) = \ln\infty - \ln e = \infty - 1 = \infty$$

The integral we were given is greater than an integral that diverges, so the given integral also $\boxed{\text{diverges}}$.

53. $\displaystyle\int_{-\infty}^{\infty} e^{-2|x|}\,dx$ $\quad |x| : \begin{cases} -x & \text{when } x < 0 \\ x & \text{when } x > 0 \end{cases}$

a) $\displaystyle\lim_{a\to-\infty} \int_a^0 e^{2x}\,dx + \lim_{b\to\infty} \int_0^b e^{-2x}\,dx$

b) $\displaystyle\lim_{a\to-\infty} \int_a^0 e^{2x}\,dx = \lim_{a\to-\infty} \frac{1}{2}e^{2x}\Big|_a^0 = \lim_{a\to-\infty} \left(\frac{1}{2}\cancel{e^0}^{(1)} - \frac{1}{2}e^{2a}\right)$

$$= \frac{1}{2} - \frac{1}{2}e^{-2\infty} = \frac{1}{2} - \frac{1}{2e^{2\infty}} = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{2} = \boxed{1}$$

$$\lim_{b\to\infty} \int_0^b e^{-2x}\,dx = \lim_{b\to\infty} -\frac{1}{2}e^{-2x}\Big|_0^b = \lim_{b\to 0} \frac{-1}{2e^{2x}}\Big|_0^b$$

$$\lim_{b\to\infty} \left(\frac{-1}{2e^{2b}} + \frac{1}{2\cancel{e^0}_{(1)}}\right) = \frac{-1}{2e^{2\infty}} + \frac{1}{2} = 0 + \frac{1}{2} = \frac{1}{2}$$