7. $f(x) = 12x^{2/3} - 4x$ (no endpts)

a) $f'(x) = 8x^{-1/3} - 4 = \frac{8}{x^{1/3}} - 4$ DNE at $x = 0$

$\frac{8}{x^{1/3}} - 4 = 0 \rightarrow \frac{8}{x^{1/3}} = 4 \rightarrow 8 = 4x^{1/3} \rightarrow x^{1/3} = 2 \rightarrow x = 8$

$f'(x)$ $- - - \overset{\text{DNE}}{|} + + \overset{0}{|} - - - - -$ $\frac{8}{x^{1/3}} - 4$

$x=0$ $\cup$ $x=8$ $\cap$

$f'(1) = \frac{8}{1} - 4 = +$

$f'(27) = \frac{8}{3} - 4 = -$

$f'(-1) = \frac{8}{-1} - 4 = -$

Inc: $\boxed{(0, 8)}$

c) $f(0) = 0 - 0 = 0 \rightarrow \boxed{(0,0) \text{ Min}}$ (sign of f' changes − to + above)

b) $f(8) = 12 \cdot 8^{2/3} - 4(8) = 12(4) - 32 = 16 \rightarrow \boxed{(8, 16) \text{ Max}}$ (sign of f' changes + to −)

d) $f'(x) = 8x^{-1/3} - 4$

$f''(x) = -\frac{8}{3}x^{-4/3} = \frac{-8}{3x^{4/3}}$ DNE at $x = 0$

$f''(x)$ $- - - \overset{\text{DNE}}{|} - - - - -$ $\frac{-8}{3x^{4/3}}$

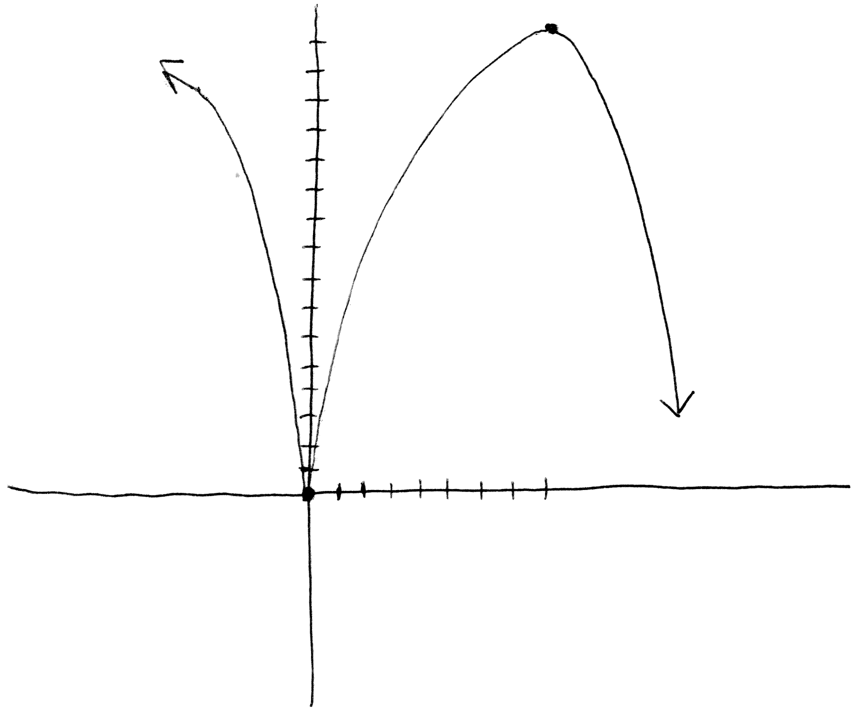
$x=0$

$f''(1) = \frac{-8}{3(1)} = -$

$f''(-1) = \frac{-8}{3(1)} = -$

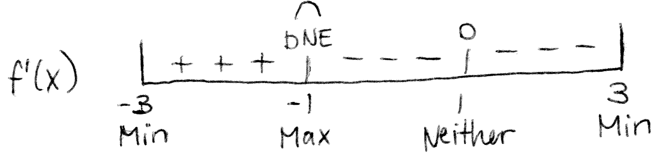
cc down: $\boxed{(-\infty, 0) \cup (0, \infty)}$

e)



9. $f(-3)=4$, $f(3)=1$, $[-3,3]$

a) $f'(x)$

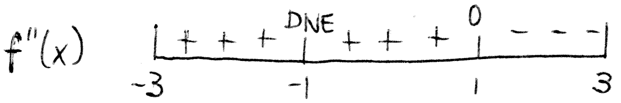


Local Min at (-3,4) bc f increases after the left endpoint

Abs. Max at x=-1 bc sign of f' changes from + to - at x=-1

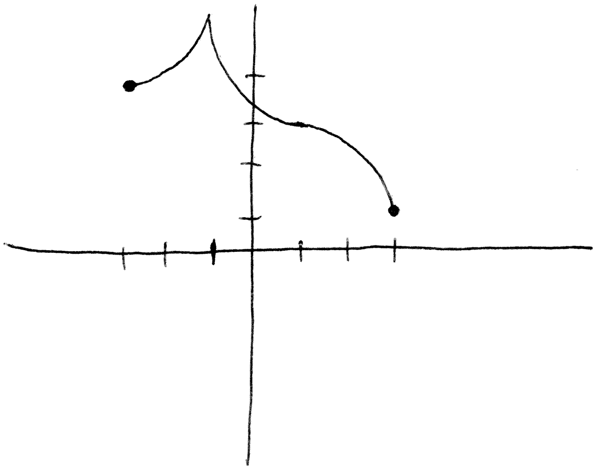
Abs. Min at (3,1) bc f decreases before the right endpoint

b) $f''(x)$



POI at x=1 bc sign of f'' changes from + to - at x=1

c)



8. $f(x) = \frac{x^3-x}{x^3-4x} = \frac{\cancel{x}(x^2-1)}{\cancel{x}(x^2-4)} = \frac{(x+1)(x-1)}{(x+2)(x-2)} = \frac{x^2-1}{x^2-4}$

a) $\lim_{x\to 0} \frac{x^3-x}{x^3-4x} = \lim_{x\to 0} \frac{(x+1)(x-1)}{(x+2)(x-2)} = \frac{(0+1)(0-1)}{(0+2)(0-2)} = \frac{-1}{-4} = \boxed{\frac{1}{4}}$

b) Zero when numerator $= 0$

$(x+1)(x-1) = 0$ at $x = 1, x = -1$ → $\boxed{(1,0) \text{ and } (-1,0)}$

c) HA : $\lim_{x\to\infty} \frac{x^3}{x^3} = 1$ → $\boxed{y=1}$

VA : Denominator $= 0$

$(x+2)(x-2) = 0$ at $\boxed{x=-2, x=2}$

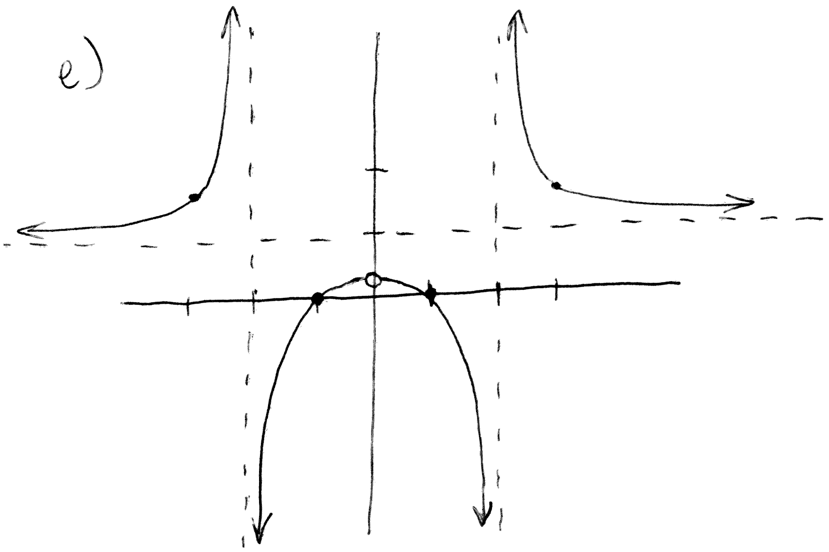
d) $\frac{x^2-1}{x^2-4}$

Example : $x = 3$ → $\frac{9-1}{9-4} = \frac{8}{5}$ Opposite x value → same y value

$f(-x) = f(x)$

$x = -3$ → $\frac{9-1}{9-4} = \frac{8}{5}$ $\boxed{\text{Even}}$ or symmetric to y-axis

e)



10. a) Rel. max when sign of f' changes + to - : $\boxed{x=-2}$

Rel. min when sign of f' changes - to + : $\boxed{x=0}$

b) f''(x) is positiven when f'(x) is increasing, so look for positive slopes of the given graph of f'(x).

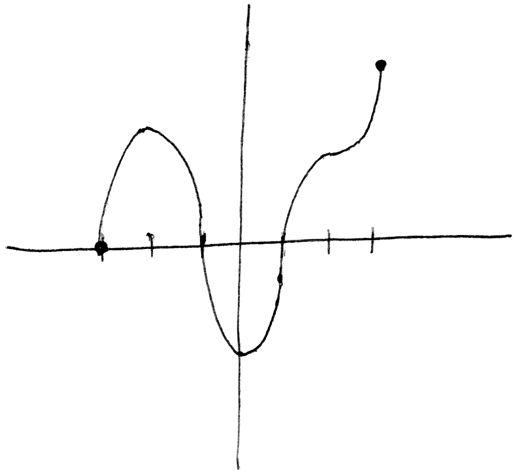
$\boxed{(-1,1) \cup (2,3)}$

c) f'(x) + + 0 - - - 0 + + 0 + +

-3 -2 0 2 3

f''(x) - - - 0 + + + 0 - - 0 + +

-3 -1 1 2 3



11. $f(x) = \frac{9x^2-36}{x^2-9} = \frac{9(x^2-4)}{x^2-9} = \frac{9(x+2)(x-2)}{(x+3)(x-3)}$

a) Example: $x=1 \rightarrow \frac{9-36}{1-9} = \frac{-27}{-8} = \frac{27}{8}$

$x=-1 \rightarrow \frac{9-36}{1-9} = \frac{-27}{-8} = \frac{27}{8}$

Opposite x value, same y value

$\boxed{\text{Even}}$ $f(-x) = f(x)$

Symmetric to y-axis

b) VA when denominator = 0 : $\boxed{x=-3, x=3}$

HA: $\lim_{x\to\infty} \frac{9x^2-36}{x^2-9} = \lim_{x\to\infty} \frac{9\cancel{x^2}}{\cancel{x^2}} = 9 \rightarrow \boxed{y=9}$

c) $f'(x) = \frac{(x^2-9)(18x)-(9x^2-36)(2x)}{(x^2-9)^2} = \frac{\cancel{18x^3}-162x-\cancel{18x^3}+72x}{(x^2-9)^2} = \frac{-90x}{(x^2-9)^2}$

$f'(x) = 0$ when $-90x = 0 \rightarrow x = 0$

$f'(x)$ DNE when $(x^2-9)^2 = 0 \rightarrow x^2-9=0 \rightarrow x^2 = 9 \rightarrow x = \pm 3$

$f'(x)$ ++ DNE +++ 0 --- DNE --- $\frac{-90x}{(x^2-9)^2}$

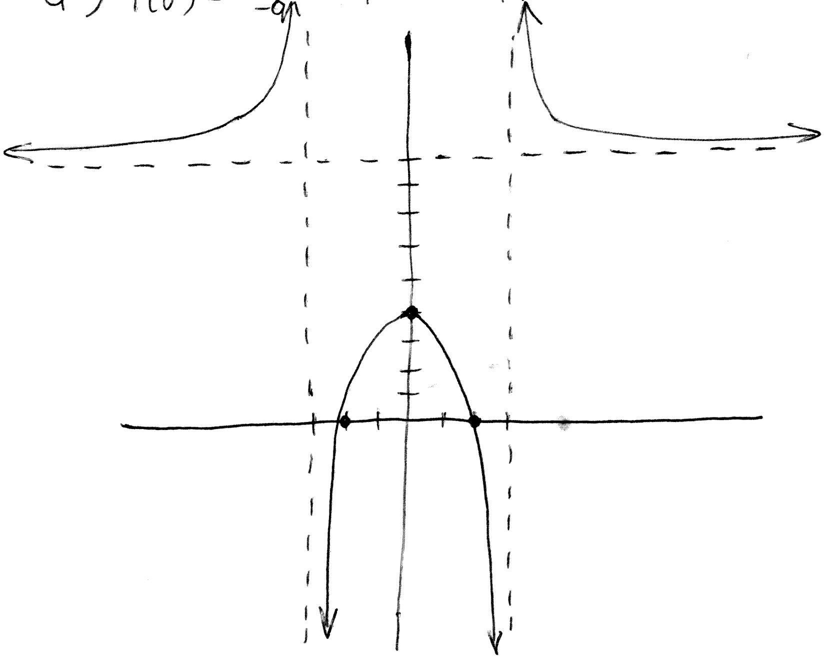
x=-3 x=0 x=3

Inc: $\boxed{(-\infty,-3) \cup (-3,0)}$

$f'(1) = \frac{-90}{64} = -$ $f'(-1) = \frac{90}{64} = +$

$f'(4) = \frac{-360}{49} = -$ $f'(-4) = \frac{360}{49} = +$

d) $f(0) = \frac{-36}{-9} = 4 \rightarrow (0,4)$



12. $f(x) = 2xe^{-x} = \frac{2x}{e^x}$

a) $\lim_{x\to\infty} \frac{2x}{e^x} = \frac{2\infty}{e^\infty}$ ← big ← much bigger $= 0 \to \boxed{y=0}$

b) No endpoints

$$f'(x) = \frac{(e^x)(2)-(2x)(e^x)}{(e^x)^2} = \frac{2e^x-2xe^x}{e^{2x}} = \frac{2\cancel{e^x}(1-x)}{\cancel{e^x}\cdot e^x} = \frac{2-2x}{e^x}$$

$f'(x) = 0$ when $2-2x=0 \to 2x=2 \to x=1$

$f'(x)$ DNE when $e^x = 0 \to$ never

$f'(x) \quad \underset{x=1}{+ + + \overset{\frown}{0} - - -} \qquad \frac{2-2x}{e^x} \to$ $\boxed{\text{Rel. max at } x=1}$

bc sign of f' changes from + to −

$f'(0) = \frac{2-2(0)}{e^0} = \frac{2}{1} = 2 = +$

$f'(2) = \frac{2-2(2)}{e^2} = \frac{-2}{e^2} = \frac{-}{+} = -$

c) $f'(x) = \frac{2-2x}{e^x}$

$$f''(x) = \frac{e^x(-2)-(2-2x)e^x}{(e^x)^2} = \frac{\cancel{e^x}(-2-2+2x)}{\cancel{e^x}\cdot e^x} = \frac{-4+2x}{e^x}$$

$f''(x) = 0$ when $-4+2x=0 \to 2x=4 \to x=2$

$f''(x)$ DNE when $e^x = 0 \to$ never

$f''(x) \quad \underset{x=2}{- - - 0 + + +} \qquad \frac{2x-4}{e^x}$

$f''(0) = \frac{-4}{e^0} = \frac{-}{+} = -$

$f''(3) = \frac{2}{e^3} = \frac{+}{+} = +$

CC down : $\boxed{(-\infty, 2)}$

d) $f(0) = \frac{2\cdot 0}{e^0} = \frac{0}{1} = 0 \to (0,0)$

$f(1) = \frac{2\cdot 1}{e^1} = \frac{2}{e} = \frac{2}{2.7} < 1 \to (1, 2/e)$

$f(2) = \frac{2\cdot 2}{e^2} = \frac{4}{\sim 8} \approx \frac{1}{2} \to (2, \sim 1/2)$