

Curve Sketching

AP Calculus Free Response Questions

****Please show all work and graph responses on a separate sheet of paper****

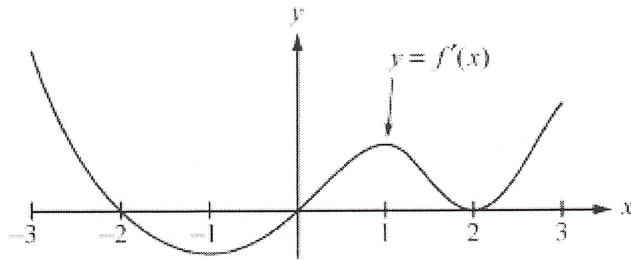
☺ NO CALCULATOR ☺

- Consider the function f given by $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$
 - Find the coordinates of all points at which the tangent to the curve is a horizontal line.
 - Find the coordinates of all points at which the tangent to the curve is a vertical line.
 - Find the coordinates of all points at which the absolute maximum and absolute minimum occur.
 - For what values of x is this function concave down?
 - Sketch the graph of the function on this interval.
- Let f be the function defined by $f(x) = |x| \cdot 0.5e^{-x^2}$ for all real numbers x .
 - Describe the symmetry of the graph of f .
 - Over what intervals of the domain is this function increasing?
 - Sketch the graph of f showing clearly:
 - behavior near the origin
 - maximum and minimum points
 - behavior for large $|x|$.
- Given the function defined by $y = x + \sin(x)$ for all x such that $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.
 - Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.
 - Find the coordinates of all points of inflection on the given interval. Justify your answers.
 - Sketch the graph of the function.
- Given the function defined by $y = e^{\sin(x)}$ for all x such that $-\pi < x < 2\pi$.
 - Find the x - and y -coordinates of all maximum and minimum points on the given interval. Justify.
 - Sketch the graph of the function.
 - Write an equation for the axis of symmetry of the graph.
- Consider the function f defined by $f(x) = (x^2 - 1)^3$ for all real numbers x .
 - For what values of x is the function increasing?
 - Find the x - and y -coordinates of the relative maximum and minimum points. Justify your answer.
 - For what values of x is the graph of f concave upward?
 - Using the information found in parts a, b, and, c, sketch the graph of f .

6. Given the function f defined by $f(x) = \cos(x) - \cos^2 x$ for $-\pi < x < \pi$.
- Find the x -intercepts of the graph of f .
 - Find the x - and y -coordinates of all relative maximum points of f . Justify your answer.
 - Find the intervals on which the graph of f is increasing.
 - Using the information found in parts a, b, and c, sketch the graph of f .
7. Let f be the function defined by $f(x) = 12x^{\frac{2}{3}} - 4x$.
- Find the intervals on which f is increasing.
 - Find the x - and y -coordinates of all relative maximum points.
 - Find the x - and y -coordinates of all relative minimum points.
 - Find the intervals on which f is concave downward.
 - Using the information found in parts a, b, c, and d, sketch the graph of f .
8. Given that f is the function defined by $f(x) = \frac{x^3 - x}{x^3 - 4x}$
- Find $\lim_{x \rightarrow 0} f(x)$
 - Find the zeros of f .
 - Write an equation for each vertical and each horizontal asymptote to the graph of f .
 - Describe the symmetry of the graph of f .
 - Using the information found in parts a, b, c, and d, sketch the graph of f .
9. A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions $f'(x)$ and $f''(x)$ have the properties given in the table below.

x	$f'(x)$	$f''(x)$
$-3 < x < -1$	Positive	Positive
$x = -1$	Fails to exist	Fails to exist
$-1 < x < 1$	Negative	Positive
$x = 1$	Zero	Zero
$1 < x < 3$	Negative	negative

- What are the x -coordinates of all absolute maximum and minimum points of f on the interval $[-3, 3]$? Justify your answer.
- What are the x -coordinates of all points of inflection on the interval $[-3, 3]$? Justify your answer.
- Sketch a graph that satisfies the given properties of f .



Note: This is the graph of the derivative of f , not the graph of f .

10. The figure above shows the graph of f' , the derivative of a function f . The domain of the function f is the set of all x such that $-3 < x < 3$.

- For what values of x , $-3 < x < 3$, does f have a relative maximum? A relative minimum? Justify.
- For what values of x is the graph of f concave up? Justify your answer.
- Use the information found in parts (a) and (b) and the fact that $f(-3) = 0$ to sketch a possible graph of f .

11. Let f be the function given by $f(x) = \frac{9x^2 - 36}{x^2 - 9}$

- Describe the symmetry of the graph of f .
- Write an equation for each vertical and each horizontal asymptote of f .
- Find the intervals on which f is increasing.
- Using the results found in parts (a), (b), and (c), sketch the graph of f .

12. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x .

- Write an equation of the horizontal asymptote for the graph of f .
- Find the x -coordinate of each critical point of f . For each such x , determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.
- For what values of x is the graph of f concave down?
- Using the results found in parts a, b, and c, sketch the graph of $y = f(x)$.