

Section 7.1: 51-64 all, 77, 79

51.  $\frac{dy}{dx} = x-1$ ,  $(1,2)$ ,  $\Delta x = 0.1$ ,  $f(1.3) = ?$

<u>Point</u>	<u><math>\frac{dy}{dx} = x-1</math></u>	<u><math>\Delta x</math></u>	<u><math>\Delta y = \frac{dy}{dx} \cdot \Delta x</math></u>	<u>New Point</u>
$(1,2)$	$1-1=0$	$0.1$	$0(0.1)=0$	$(1.1, 2)$
$(1.1, 2)$	$1.1-1=0.1$	$0.1$	$0.1(0.1)=0.01$	$(1.2, 2.01)$
$(1.2, 2.01)$	$1.2-1=0.2$	$0.1$	$0.2(0.1)=0.02$	$(1.3, \boxed{2.03})$

52.  $\frac{dy}{dx} = y-1$ ,  $(1,3)$ ,  $\Delta x = 0.1$ ,  $f(1.3) = ?$

$(1,3)$	$3-1=2$	$0.1$	$2(0.1)=0.2$	$(1.1, 3.2)$
$(1.1, 3.2)$	$3.2-1=2.2$	$0.1$	$2.2(0.1)=0.22$	$(1.2, 3.42)$
$(1.2, 3.42)$	$3.42-1=2.42$	$0.1$	$2.42(0.1)=0.242$	$(1.3, \boxed{3.662})$

53.  $\frac{dy}{dx} = y-x$ ,  $(1,2)$ ,  $\Delta x = 0.1$ ,  $f(1.3) = ?$

$(1,2)$	$2-1=1$	$0.1$	$1(0.1)=0.1$	$(1.1, 2.1)$
$(1.1, 2.1)$	$2.1-1.1=1$	$0.1$	$1(0.1)=0.1$	$(1.2, 2.2)$
$(1.2, 2.2)$	$2.2-1.2=1$	$0.1$	$1(0.1)=0.1$	$(1.3, \boxed{2.3})$

54.  $\frac{dy}{dx} = 2x-y$ ,  $(1,0)$ ,  $\Delta x = 0.1$ ,  $f(1.3) = ?$

$(1,0)$	$2(1)-0=2$	$0.1$	$2(0.1)=0.2$	$(1.1, 0.2)$
$(1.1, 0.2)$	$2(1.1)-0.2=2$	$0.1$	$2(0.1)=0.2$	$(1.2, 0.4)$
$(1.2, 0.4)$	$2(1.2)-0.4=2$	$0.1$	$2(0.1)=0.2$	$(1.3, \boxed{0.6})$

55.  $\frac{dy}{dx} = 2-x$ ,  $(2,1)$ ,  $\Delta x = -0.1$ ,  $f(1.7) = ?$

$(2,1)$	$2-2=0$	$-0.1$	$0(-0.1)=0$	$(1.9, 1)$
$(1.9, 1)$	$2-1.9=0.1$	$-0.1$	$0.1(-0.1)=-0.01$	$(1.8, 0.99)$
$(1.8, 0.99)$	$2-1.8=0.2$	$-0.1$	$0.2(-0.1)=-0.02$	$(1.7, \boxed{0.97})$

56.  $\frac{dy}{dx} = 1+y$ ,  $(2,0)$ ,  $\Delta x = -0.1$ ,  $f(1.7) = ?$

Point	$\frac{dy}{dx} = 1+y$	$\Delta x$	$\Delta y = \frac{dy}{dx} \cdot \Delta x$	New Point
$(2,0)$	$1+0=1$	$-0.1$	$1(-0.1)=-0.1$	$(1.9, -0.1)$
$(1.9, -0.1)$	$1-0.1=0.9$	$-0.1$	$0.9(-0.1)=-0.09$	$(1.8, -0.19)$
$(1.8, -0.19)$	$1-0.19=0.81$	$-0.1$	$0.81(-0.1)=-0.081$	$(1.7, \boxed{-0.271})$

57.  $\frac{dy}{dx} = x-y$ ,  $(2,2)$ ,  $\Delta x = -0.1$ ,  $f(1.7) = ?$

$(2,2)$	$2-2=0$	$-0.1$	$0(-0.1)=0$	$(1.9, 2)$
$(1.9, 2)$	$1.9-2=-0.1$	$-0.1$	$-0.1(-0.1)=0.01$	$(1.8, 2.01)$
$(1.8, 2.01)$	$1.8-2.01=-0.21$	$-0.1$	$-0.21(-0.1)=0.021$	$(1.7, \boxed{2.031})$

58.  $\frac{dy}{dx} = x-2y$ ,  $(2,1)$ ,  $\Delta x = -0.1$ ,  $f(1.7) = ?$

$(2,1)$	$2-2(1)=0$	$-0.1$	$0(-0.1)=0$	$(1.9, 1)$
$(1.9, 1)$	$1.9-2(1)=-0.1$	$-0.1$	$-0.1(-0.1)=0.01$	$(1.8, 1.01)$
$(1.8, 1.01)$	$1.8-2.02=-0.22$	$-0.1$	$-0.22(-0.1)=0.022$	$(1.7, \boxed{1.032})$

59. a) Graph B

b)  $\frac{dy}{dx} = \frac{1}{1+x^2}$  is always positive, so the solution always increases.

60. a) Graph B

b)  $\frac{dy}{dx} = -x$  is positive when  $x < 0$ , so the solution increases when  $x < 0$ .

$\frac{dy}{dx} = -x$  is negative when  $x > 0$ , so the solution decreases when  $x > 0$ .

61.  $y = x^2$  cannot be a solution because  $y = x^2$  would have negative slopes in the second quadrant, while the slope field shows positive slopes in the second quadrant.

62.  $y = \sin x$  cannot be a solution because  $y = \sin x$  would have slopes of 1 anywhere  $x = 0$ , but the slope field shows slopes of 0 anywhere  $x = 0$ .

63.  $\frac{dy}{dx} = 2x+1$ ,  $(1,3)$ ,  $\Delta x = 0.1$ ,  $f(1.4) = ?$

Point	$\frac{dy}{dx} = 2x+1$	$\Delta x$	$\Delta y = \frac{dy}{dx} \cdot \Delta x$	New Point
$(1,3)$	$2(1)+1 = 3$	0.1	$3(0.1) = 0.3$	$(1.1, 3.3)$
$(1.1, 3.3)$	$2(1.1)+1 = 3.2$	0.1	$3.2(0.1) = 0.32$	$(1.2, 3.62)$
$(1.2, 3.62)$	$2(1.2)+1 = 3.4$	0.1	$3.4(0.1) = 0.34$	$(1.3, 3.96)$
$(1.3, 3.96)$	$2(1.3)+1 = 3.6$	0.1	$3.6(0.1) = 0.36$	$(1.4, 4.32)$

$y = x^2 + x + C \rightarrow 3 = 1 + 1 + C \rightarrow C = 1 \rightarrow y = x^2 + x + 1$

Actual:  $y(1.4) = 4.36$

% error =  $\frac{4.36 - 4.32}{4.36} \times 100\% = \boxed{0.917\%}$

64.  $\frac{dy}{dx} = 2x-1$ ,  $(2,3)$ ,  $\Delta x = -0.1$ ,  $f(1.6) = ?$

$(2,3)$	$2(2)-1 = 3$	-0.1	$3(-0.1) = -0.3$	$(1.9, 2.7)$
$(1.9, 2.7)$	$2(1.9)-1 = 2.8$	-0.1	$2.8(-0.1) = -0.28$	$(1.8, 2.42)$
$(1.8, 2.42)$	$2(1.8)-1 = 2.6$	-0.1	$2.6(-0.1) = -0.26$	$(1.7, 2.16)$
$(1.7, 2.16)$	$2(1.7)-1 = 2.4$	-0.1	$2.4(-0.1) = -0.24$	$(1.6, 1.92)$

$y = x^2 - x + C \rightarrow 3 = 4 - 2 + C \rightarrow C = 1 \rightarrow y = x^2 - x + 1$

Actual:  $y(1.6) = 1.96$

% error =  $\frac{1.96 - 1.92}{1.96} \times 100\% = \boxed{2.041\%}$

77. a)  $y'' = 12x + 4$

$y' = 6x^2 + 4x + C_1$

$y = \boxed{2x^3 + 2x^2 + C_1x + C_2}$

b)  $y'' = e^x + \sin x$

$y' = e^x - \cos x + C_1$

$y = \boxed{e^x - \sin x + C_1x + C_2}$

c)  $y'' = x^3 + x^{-3}$

$y' = \frac{1}{4}x^4 - \frac{1}{2}x^{-2} + C_1$

$y = \boxed{\frac{1}{20}x^5 + \frac{1}{2}x^{-1} + C_1x + C_2}$

$$79. a) \frac{dy}{dx} = x \rightarrow y = \boxed{\frac{1}{2}x^2 + C}$$

$$b) \frac{dy}{dx} = -x \rightarrow y = \boxed{-\frac{1}{2}x^2 + C}$$

$$c) \frac{dy}{dx} = y$$

$$\int \frac{1}{y} dy = \int 1 dx$$

$$\ln|y| = x + C$$

$$\boxed{y = Ce^x}$$

$$d) \frac{dy}{dx} = -y$$

$$\int \frac{1}{y} dy = \int -1 dx$$

$$\ln|y| = -x + C$$

$$\boxed{y = Ce^{-x}}$$

$$e) \frac{dy}{dx} = xy$$

$$\int \frac{1}{y} dy = \int x dx$$

$$\ln|y| = \frac{1}{2}x^2 + C$$

$$\boxed{y = Ce^{\frac{1}{2}x^2}}$$