Numerical Method / Euler's Method (Section 7.1)

* Euler's Method (pronounced "Oilers") is used to approximate a curve using local linearity.

STEPS

① Must have an initial condition

② Use $\frac{dy}{dx}$ to find the slope at that point.

③ Increase x by a small amount (Δx) to obtain a new point $(x+\Delta x, y+\Delta y)$

* $\Delta y = \frac{dy}{dx} \cdot \Delta x$

④ Keep repeating steps 2 & 3 until you reach the x-coordinate you are looking for.

* If Δx is $(+)$, you're moving to the right
  If Δx is $(-)$, you're moving to the left.

ex: $\frac{dy}{dx} = x+y$ $\quad f(2)=0$ $\quad$ Use Euler's method & $\Delta x = .2$ to approx $f(3)$.

* TABLE METHOD:

Point	$\frac{dy}{dx} = x+y$	Δx	$\Delta y = \frac{dy}{dx} \cdot \Delta x$	New Point: $(x+\Delta x, y+\Delta y)$
$(2,0)$	$2+0=2$	$.2$	$2(.2)=.4$	$(2.2, .4)$
$(2.2, .4)$	$2.2+.4=2.6$	$.2$	$2.6(.2)=.52$	$(2.4, .92)$
$(2.4, .92)$	$2.4+.92=3.32$	$.2$	$3.32(.2)=.664$	$(2.6, 1.584)$
$(2.6, 1.584)$	$2.6+1.584=4.184$	$.2$	$4.184(.2)=.8368$	$(2.8, 2.4208)$
$(2.8, 2.4208)$	$2.8+2.4208=5.2208$	$.2$	$5.2208(.2)=1.04416$	$(3.0, 3.46496)$

$$\boxed{\therefore f(3) \approx 3.465}$$

* Equation of a line method: $y_1 = y_0 + f'(x_0, y_0)dx$

ex: $y' = x - y$

$y(0) = 1$

$dx = .1$

Approx $f(.3)$

$(0,1)$ $\quad y_1 = 1 + (0-1)(.1)$
$= 1 - .1 = .9$

$(.1,.9)$ $\quad y_2 = .9 + (.1-.9)(.1)$
$= .9 - .08 = .82$

$(.2,.82)$ $\quad y_3 = .82 + (.2-.82)(.1)$
$= .82 - .062 = .758$

$(.3,.758)$ $\quad \boxed{\therefore\ f(.3) \approx .758}$

** Use which ever method makes sense to you!!!

Line method

#64 $\frac{dy}{dx} = 2x - 1$

$f(2) = 3$

$\Delta x = -.1$

$f(1.6) = ?$

a) $(2,3)$ $\quad y_1 = 3 + [2(2)-1](-.1) = 3 - .3 = 2.7$

$(1.9, 2.7)$ $\quad y_2 = 2.7 + [2(1.9)-1](-.1) = 2.7 - .28 = 2.42$

$(1.8, 2.42)$ $\quad y_3 = 2.42 + [2(1.8)-1](-.1) = 2.42 - .26 = 2.16$

$(1.7, 2.16)$ $\quad y_4 = 2.16 + [2(1.7)-1](-.1) = 2.16 - .24 = 1.92$

$(1.6, 1.92)$ $\quad \boxed{\therefore\ f(1.6) \approx 1.92}$

b) $y = x^2 - x + c \rightarrow 3 = 2^2 - 2 + c$
$c = 1$

$y = x^2 - x + 1$

$f(1.6) = 1.96$ $\quad$ % error $= \frac{1.96 - 1.92}{1.96} = .02$

$\boxed{\approx 2\% \text{ error}}$

#77 a) $\int y'' = \int 12x + 4$

$\int y' = \int 6x^2 + 4x + c$

$\boxed{y = 2x^3 + 2x^2 + c_1 x + c_2}$