

Implicit Differentiation Practice

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

1) $x = 2y^2 + 4$

$$1 = 4y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \boxed{\frac{1}{4y}}$$

2) $2 = 5x - y^2$

$$0 = 5 - 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \boxed{\frac{5}{2y}}$$

3) $3x^3 = 3x^3y + 1$

$$9x^2 = 3x^3 \frac{dy}{dx} + y \cdot 9x^2$$

$$\frac{dy}{dx} = \frac{9x^2 - 9x^2y}{3x^3} = \boxed{\frac{3x^2 - 3x^2y}{x^3}}$$

4) $x = -3xy^2 + 2$

$$1 = -3x \cdot 2y \frac{dy}{dx} + y^2 \cdot -3$$

$$\frac{dy}{dx} = \boxed{\frac{1 + 3y^2}{-6xy}}$$

5) $3x^2 = -5y + 5x^3y$

$$6x = -5 \frac{dy}{dx} + 5x^3 \frac{dy}{dx} + y \cdot 15x^2$$

$$\frac{dy}{dx} (5 - 5x^3) = 15x^2y - 6x$$

$$\frac{dy}{dx} = \boxed{\frac{15x^2y - 6x}{5 - 5x^3}}$$

6) $4x + 5x^2y = 3y^2$

$$4 + 5x^2 \frac{dy}{dx} + y \cdot 10x = 6y \frac{dy}{dx}$$

$$\frac{dy}{dx} (5x^2 - 6y) = -10xy - 4$$

$$\frac{dy}{dx} = \boxed{\frac{-10xy - 4}{5x^2 - 6y}}$$

$$7) (2y^3 + 1)^2 = 5x$$

$$2(2y^3 + 1) \cdot 6y^2 \frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = \frac{5}{12y^2(2y^3 + 1)} = \boxed{\frac{5}{24y^5 + 12y^2}}$$

$$8) 3x^2 = (2y^3 + 5)^2$$

$$6x = 2(2y^3 + 5) \cdot 6y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{6x}{2 \cdot 6y^2(2y^3 + 5)} = \frac{x}{2y^2(2y^3 + 5)} = \boxed{\frac{x}{4y^5 + 10y^2}}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y .

$$9) 2x^3 = 3y^2 + 1$$

$$6x^2 = 6y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{6x^2}{6y} = \boxed{\frac{x^2}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{y \cdot 2x - x^2 \cdot \frac{dy}{dx}}{y^2}$$

$$= \frac{2xy - x^2 \cdot \frac{x^2}{y}}{y^2}$$

$$= \frac{2xy^2 - x^4}{y^3}$$

$$= \frac{2xy^2 - x^4}{y} \cdot \frac{1}{y^2} = \boxed{\frac{2xy^2 - x^4}{y^3}}$$

$$10) x = -4y^2 + 4$$

$$1 = -8y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-1}{8y} = -\frac{1}{8} y^{-1}$$

$$\frac{d^2y}{dx^2} = \frac{1}{8} y^{-2} \frac{dy}{dx} = \frac{1}{8y^2} \cdot \frac{-1}{8y} = \boxed{\frac{-1}{64y^3}}$$