Linearization & Newton's Method (Section 5.5)

* **Linearization:** If f is differentiable @ $x=a$, then the equation of the tangent line:

$$\boxed{L(x) = f(a) + f'(a)(x-a)}$$

defines the linearization of $f(x)$ @ $x=a$, where "a" is the center of the approximation.

on HW: #1-8

ex: $f(x)=\sqrt{1+x}$ @ $x=\overset{a}{0}$; Approx: $\sqrt{1.02}$

* Approximating Binomial Powers: for SMALL values of x:

$$(1+x)^k \approx 1+k\cdot x$$

on HW #9-10

ex: $\sqrt{1+5x}$

ex: $\sqrt[3]{1-x}$

on HW #27-30

ex: $f(x) = x^3 - x$; $a = 1$; $dx = 0.1$ Find the true change, est change, and error.

* Newton's Method: A numerical technique of approximating a zero of a function.

STEPS: ① Guess a first approximation of $f(x)=0$ (look @ the graph)

② Use: Newton's Method

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

The 1ST approx finds the 2nd the 2nd approx finds the 3rd...

③ Keep going until the approximation doesn't change.

IN CALCULATOR*

$f(x) \to y_1$

$f'(x) \to y_2$

Then use:

$$\underbrace{x_{n+1} = ANS - \frac{y_1(ANS)}{y_2(ANS)}}_{\text{Put this into the Calculator}}$$

* Newton's method does not work if $f'(x_1) = 0$.

* Newton's method must converge to find a zero → if successive approx go back & forth between 2 values, it will not converge!

* starting value must be close to the zero you want to find.

on HW #53-56

ex: Use Newton's method to solve: $f(x) = x^3 + 3x + 1 = 0$.

* Approximating Roots: decide where to center the approximation & then linearize the function

HW #11-14

ex: $\sqrt{123}$ OR $\sqrt[3]{123}$

* Differentials: Let $y = f(x)$ be differentiable function. The differential "dx" is an independent variable & $dy = f'(x) \cdot dx$

on HW #15-22

ex: $y = x^5 + 37x$; $x = 1$; $dx = .01$

ex: $x + y = x \cdot y$; $x = 2$; $dx = .05$

* The differential of f is df → basically the derivative w/dx on the end.

on HW #23-26

ex: $f(x) - 3x^2 - 6$