Logs and Exponents Worksheet - Key

1) $\frac{1}{4^{x-2}} = 64$

$4^{x-2} = \frac{1}{64}$

$4^{-3} = \frac{1}{64}$, so $x-2 = -3$

$\boxed{x = -1}$

2) $5^{3x-1} = 4^0$

$5^{3x-1} = 1$

$5^0 = 1$, so $3x-1 = 0$

$\boxed{x = 1/3}$

3) $\log_2(\ln x) = 2$

$2^2 = \ln x$

$4 = \ln_e x$

$\boxed{e^4 = x}$

4) $2\log x = 4$

$\log x = 2$

$\log_{10} x = 2$

$10^2 = x$

$\boxed{100 = x}$

5) $2^x = 5$

$x \cdot \log 2 = \log 5$

$x = \frac{\log 5}{\log 2} = \boxed{\log_2 5}$

6) $\log_3(2x) - \log_3(x+5) = 0$

$\log_3\left(\frac{2x}{x+5}\right) = 0$

$3^0 = \frac{2x}{x+5}$

$1 = \frac{2x}{x+5}$

$x+5 = 2x$

$\boxed{x = 5}$

7) $\log(x^3-1) - \log(x^2+x+1) = 1$

$\log_{10}\left(\frac{x^3-1}{x^2+x+1}\right) = 1$

$10^1 = \frac{x^3-1}{x^2+x+1}$

$10 = \frac{(x-1)(x^2+x+1)}{(x^2+x+1)}$

$10 = x-1$

$\boxed{x = 11}$

8) $\log_{12}(x+5) + \log_{12}(x-5) = 2$

$\log_{12}(x^2-25) = 2$

$12^2 = x^2-25$

$144 = x^2-25$

$169 = x^2$

$\boxed{x = \pm 13}$

9) $x^{3/5} = 8$

$(x^{3/5})^{5/3} = 8^{5/3}$

$x = (\sqrt[3]{8})^5 = 2^5 = \boxed{32}$

10) $\ln(x+2) = \ln x + \ln 2$

$\ln(x+2) = \ln(2x)$

$x+2 = 2x$

$\boxed{x = 2}$

11) $2\ln(x+1) = 4\ln 5$

$\ln(x+1) = 2\ln 5$

$\ln(x+1) = \ln 5^2$

$x+1 = 25$

$\boxed{x = 24}$

12) $\sqrt[6]{81} = (81)^{1/6} = (3^4)^{1/6}$

$\sqrt[6]{81} = 3^{4/6} = \boxed{3^{2/3}}$

13) $7^{-2} \cdot 27^{1/3} = \dfrac{\sqrt[3]{27}}{7^2} = \boxed{\dfrac{3}{49}}$

14) $\left(\dfrac{-8a^3}{b^{-6}}\right)^{2/3} = \dfrac{(\sqrt[3]{-8})^2 a^2}{b^{-4}} = \boxed{4a^2b^4}$

15) $e^{\ln 7} = \boxed{7}$

16) $\dfrac{\log_5 9}{\log_5 3} = \dfrac{\left(\frac{\log 9}{\log 5}\right)}{\left(\frac{\log 3}{\log 5}\right)} = \dfrac{\log 9}{\log 3}$

$\dfrac{\log_5 9}{\log_5 3} = \log_3 9$

$\log_3 9 = x$

$3^x = 9$, so $\boxed{x = 2}$

17) $\log_4 64^{3/4} = x$

$4^x = 64^{3/4}$

$4^x = (4^3)^{3/4}$

$4^x = 4^{9/4}$

$\boxed{x = 9/4}$

18) $500(125)^{-2/3} = \dfrac{500}{(\sqrt[3]{125})^2}$

$= \dfrac{500}{5^2} = \dfrac{500}{25} = \boxed{20}$

19) $\sqrt[4]{\sqrt[4]{64}} \cdot \sqrt[16]{16} = (64^{1/4})^{1/4} \cdot 16^{1/16} = 64^{1/16} \cdot 16^{1/16}$

$= (2^6)^{1/16} \cdot (2^4)^{1/16} = 2^{6/16} \cdot 2^{4/16} = 2^{10/16} = \boxed{2^{5/8}}$

20) $\log_3 40 - \log_3 8$

$= \log_3\left(\frac{40}{8}\right)$

$= \log_3(5) = \boxed{\log_3 5}$

21) $\log_5 7 + \log_5 4$

$= \log_5(7 \cdot 4)$

$= \boxed{\log_5 28}$

22) $3 + \log_3 4$

↑ We want 3 to also be written as a log with base 3.

$\log_3 x = 3$

$3^3 = x$

$x = 27$

$\log_3 27 + \log_3 4 = \log_3(27 \cdot 4) = \boxed{\log_3 108}$