Name _____

M-07 Functions 1 : Basics

Answer on this sheet

1. The following functions are given:

$f(x) = 3x - 2$	$g(x) = x^2 - 4$
$h(x) = \sqrt{2x-3}$	$k(x) = \dfrac{10-x}{3}$

(a) Find

(i) $f(4) =$ (ii) $g(6) =$

(iii) $h(26) =$ (iv) $k(-2) =$

(v) $f(-\frac{1}{6}) =$ (vi) $g(-3.5) =$

(vii) $k(0.1) =$ (viii) $g(3) - f(3) =$

(ix) $h(1) =$ (x) $k(-3.2)$

(b) Find the value of x if:

(i) $k(x) = 7$ (ii) $h(x) = 4$

(iii) $f(x) = x$ (iv) $k(x) = f(x)$

(v) $g(x) \times h(x) = 0$

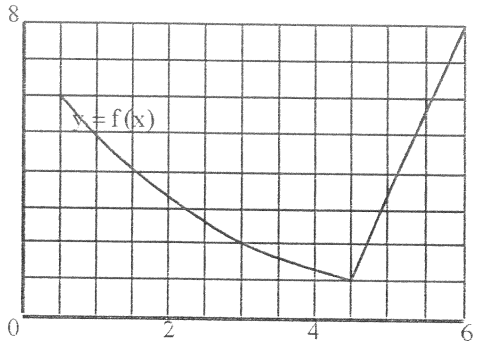
2. Using the functions from Q1, find

(a) $k(k(4)) =$

(b) $h(k(-8)) =$

(c) $f(f(f(1))) =$

3.



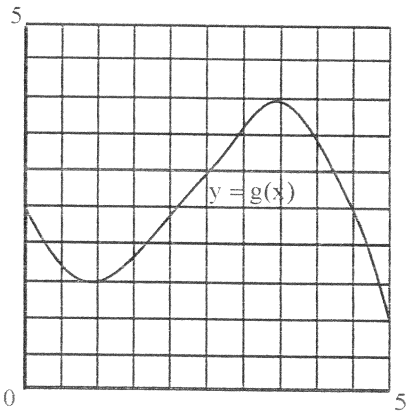
(a) Write down the domain and range of the function graphed above.

D: R:

(b) Find $f(3) =$

(c) Solve $f(x) = 4$ $x =$

4.



(a) Solve $g(x) = 3$ $x =$

(b) By drawing a line on the graph solve $g(x) = x$

$x =$

(c) Write down a positive integer k such that the equation $g(x) = k$ has three solutions in the given domain.

$k =$

5. (a) Find the range of $g(x) = 10 - 3x$ for the domain $0 \le x \le 3$.

(b) Find the range of $h(x) = x^2 - 4x + 6$ for the domain $0 \le x \le 3$.

M-08 Functions 2 : Composite and Inverse

1. Given these functions:

$$f : x \to 3x - 2$$
$$g : x \to \frac{x}{x-2}$$
$$h : x \to x(6-x)$$

find the values of the following:

(a) $f \circ g(3)$ (b) $g \circ h(5)$

(c) $f \circ h(2)$ (d) $f \circ f(5)$

(e) $g \circ g(-4)$ (f) $h \circ g \circ f(2)$

(g) $h \circ g(1)$ (h) $g \circ h(1)$

2. Find the inverse function in each case:

(a) $f(x) = 4x - 1$

(b) $f(x) = 3(x-2)$

(c) $f(x) = \frac{x+2}{5}$

(d) $f(x) = 2(1-2x)$

(e) $f(x) = \frac{10-2x}{3}$

(f) $f(x) = \sqrt{x-5} \quad (x \geq 5)$

(g) $f(x) = 1 + x^3$

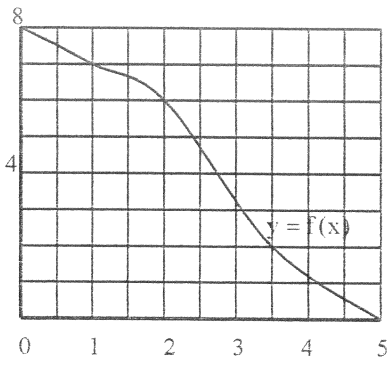
3. $h(x) = \frac{4-x}{3}$

(a) Find $h^{-1}(x)$.

(b) Use the answer to (a) to solve $h^{-1}(x) = -2$.

(c) Find another (simpler!) way to solve $h^{-1}(x) = -2$.

4.



(a) Find (i) $f(2) =$ (ii) $f^{-1}(2) =$

(b) Find (i) $f^{-1}(7) =$ (ii) $f(f^{-1}(7)) =$

(c) Solve $f^{-1}(x) = 3$ $x =$

(d) Find $f \circ f(2.5)$ as accurately as you can:

$f \circ f(2.5) =$

5. (a) Given $f(x) = 2x + 1$ and $g(x) = 5x$, find $f \circ g(x)$.

(b) Given $f(x) = 1 - 3x$ and $g(x) = 2x + 7$, find $g \circ f(x)$.

(c) Given $f(x) = \frac{x}{2} + 1$ and $g(x) = 6 - 2x$, find $f \circ g(x)$.

6. Given the functions

$$f(x) = 3x - 2 \qquad g(x) = \frac{x+2}{5}$$

find:

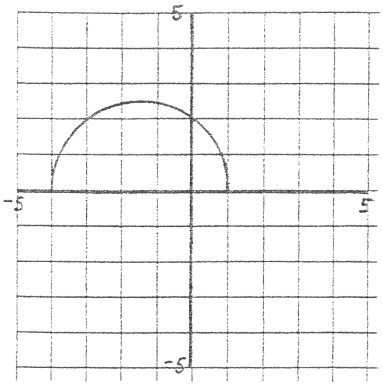
(a) $f \circ g(x)$, giving your answer in the form $f \circ g(x) = \frac{ax+b}{c}$ (with $a, b, c \in \mathbb{Z}$)

(b) $f \circ (g^{-1})(x)$

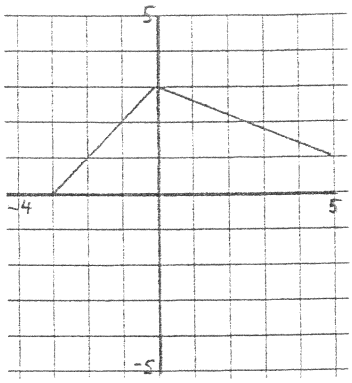
M-09 **Transforming Graphs 1 : Translations, Stretches**

In each diagram the below the given graph is $y = f(x)$. Sketch the transformation of $y = f(x)$ indicated in brackets.

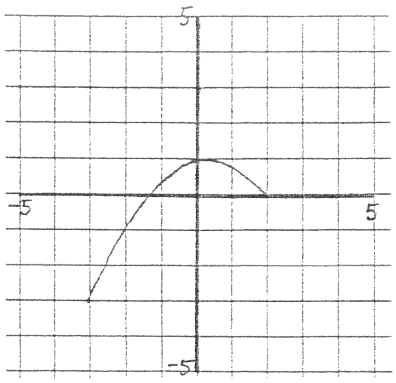
(a) $[y = f(x-2)]$



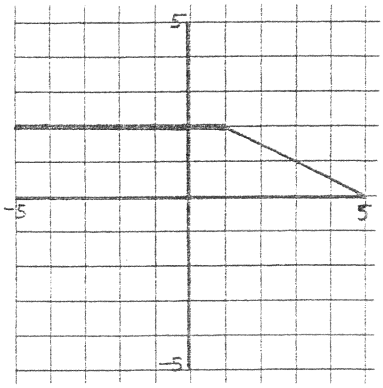
(b) $[y = f(x+1)]$



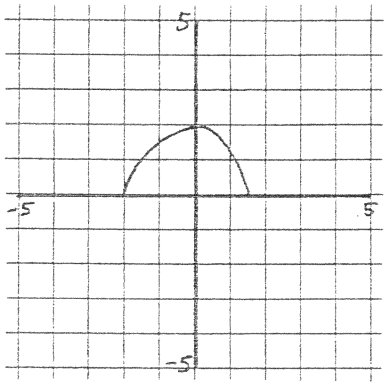
(c) $[y = f(x)+3]$



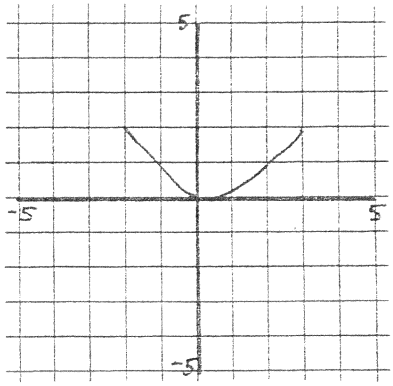
(d) $[y = 2f(x)]$



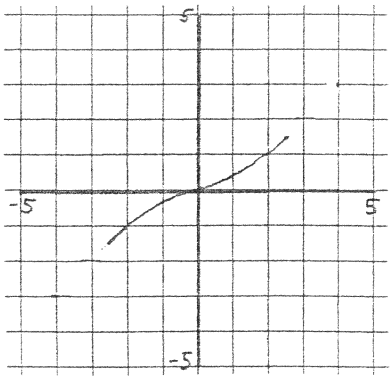
(e) $[y = f(2x)]$



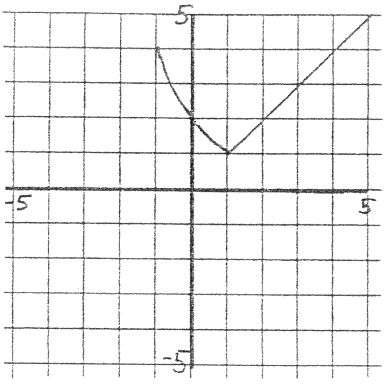
(f) $[y = f(x-1)-3]$



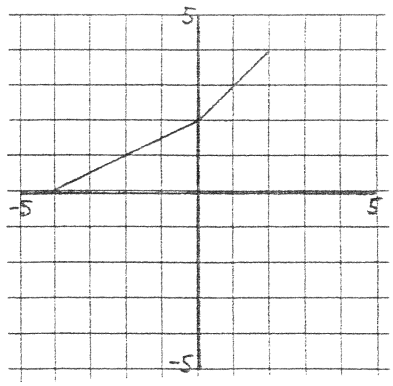
(g) $\left[y = f\left(\frac{x}{2}\right)\right]$



(h) $\left[y = \frac{f(x)}{2}\right]$



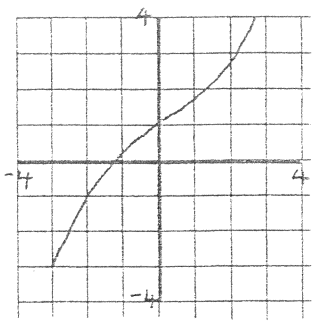
(i) $[y = f(2x)-3]$



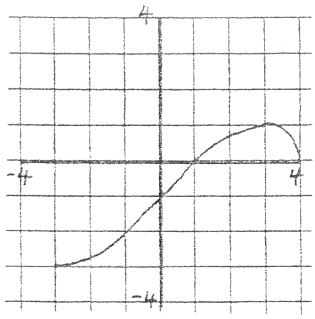
M-10 Transforming Graphs 2 : Reflections, Mixed

1. In each diagram below the given graph is $y = f(x)$. Sketch the transformation of $y = f(x)$ indicated in brackets.

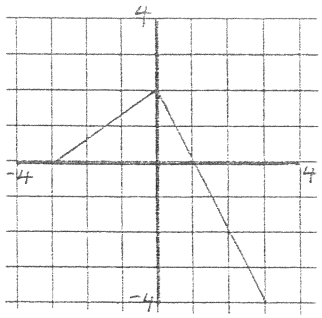
(a) $[y = -f(x)]$



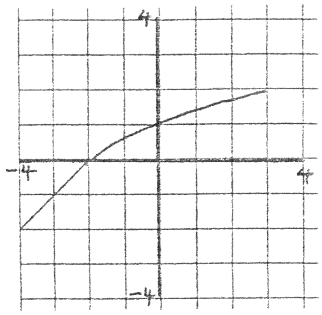
(b) $[y = f(-x)]$



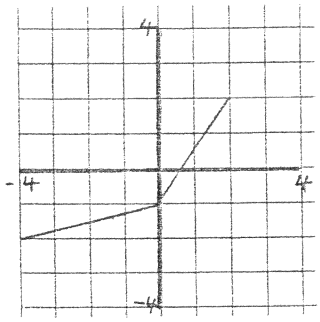
(c) $[y = -f(-x)]$



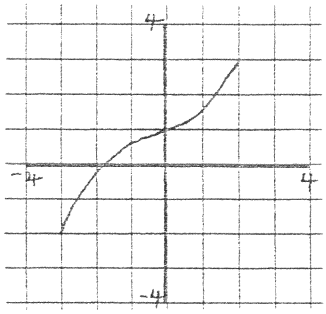
(d) $[y = -f(x) + 2]$



(e) $[y = -2f(x)]$



(f) $[y = f(-x) - 2]$



2. Write down the transformation of $y = f(x)$ represented by the graphs labeled:

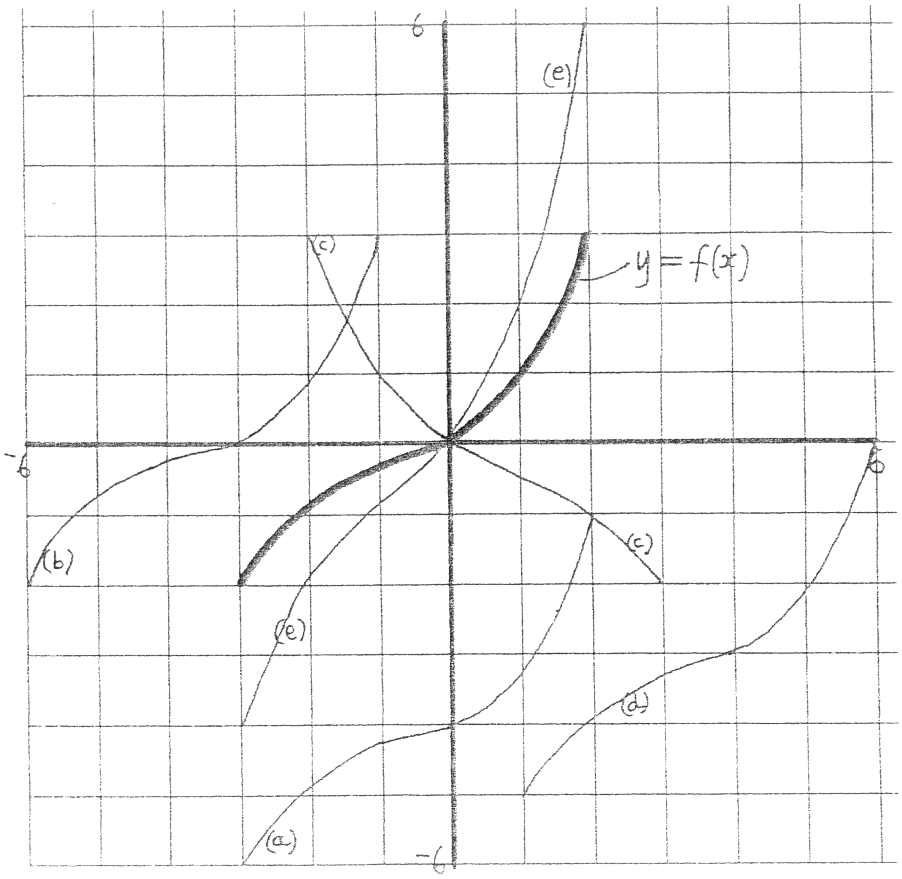
(a)

(b)

(c)

(d)

(e)



M-11 The Quadratic Function

1. Write each quadratic in the form $a(x-h)^2+k$:

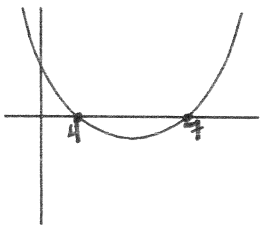
   (a) $y = x^2 + 6x + 10$
   (b) $y = x^2 - 4x - 7$
   (c) $y = x^2 + 3x$
   (d) $y = x^2 - 8x + 10$

   (e) $y = (x - 2)(x + 6)$
   (f) $y = x^2 + 5x + 1$
   (g) $y = 2x^2 + 8x + 7$
   (h) $y = 1 + 2x - 2x^2$

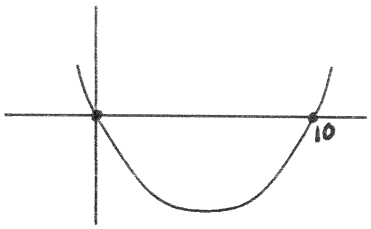
2. Write down the axis of symmetry of each quadratic in Q1. $\rightarrow x = \frac{-b}{2a}$

3. Each sketch graph below represents a quadratic of the form $y = x^2 + bx + c$. Find the values of b and c in each case:

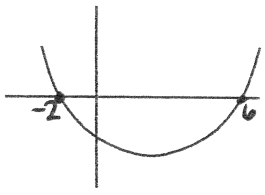
   (a)



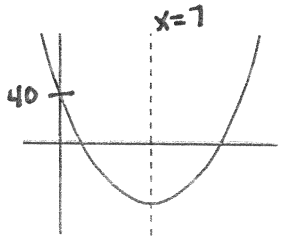
   (b)



   (c)

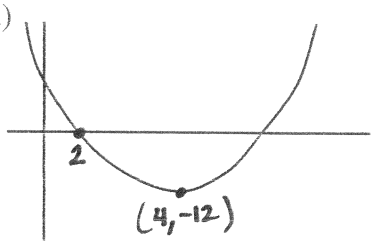


   (d)

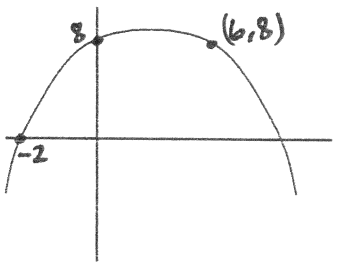


4. Each sketch graph below represents a quadratic of the form $y = ax^2 + bx + c$. Find the values of a, b and c in each case:

   (a)



   (b)



Sketches may help with these :

5. A quadratic has its vertex at (3, 2) and passes through (5, 14)

   (a) For what other x-value will the y-value = 14?

   (b) Find the equation of the quadratic in the form $y = ax^2 + bx + c$.

6. A quadratic which passes through (2, 8) has a zero at x = 4. Its y-intercept is 12.

   Find (a) the equation of the quadratic in the form $y = ax^2 + bx + c$

   (b) the other zero of the quadratic.

7. A quadratic of the form $y = ax^2 + bx$ has a line of symmetry at x = 2.5. Find the zeros of the quadratic.

8. The table for $y = ax^2 + bx + c$ is shown. Find the values of a, b and c.

x	0	1	2
y	-5	0	9

1. Solve these quadratic equations using a factorisation method:

   (a) $x^2 + 4x - 32 = 0$
   (b) $x^2 - 3x - 18 = 0$
   (c) $x^2 + x - 42 = 0$
   (d) $2x^2 + x - 6 = 0$

   (e) $x^2 = 5x$
   (f) $x^2 + x = 30$
   (g) $3x^2 = 2x + 40$
   (h) $\dfrac{x}{x-4} = x - 3$

   (i) $2x(x-3) = 20$
   (j) $(x+2)^2 = 8x + 1$
   (k) $41 = (x+1)^2 + (x+2)^2$
   (l) $\dfrac{60}{x-1} - 2 = x + 4$

2. Where possible, find the zeros **correct to two decimal places**, using the quadratic formula or otherwise:

   (a) $x^2 - 2x - 20 = 0$
   (b) $2x^2 - 3x - 11 = 0$
   (c) $x^2 + x = 1000$

   (d) $\dfrac{20}{x-3} = x + 1$
   (e) $x^2 = 6x - 10$
   (f) $(x-4)^2 = 7x$

3. (a) Write down a quadratic equation which has roots $x = 5$ and $x = -7$.

   (b) Write down a quadratic equation which has a repeated root $x = 3$.

   (c) Write down (in the form $ax^2 + bx + c = 0$ with $a, b, c \in \mathbb{Z}$) a quadratic equation which has roots $x = \dfrac{1}{3}$ and $x = -\dfrac{1}{2}$.

   (d) $x^2 + bx + c = 0$ has a repeated root $x = -7$. Find the values of b and c.

4. In this question we use:
   T to mean "the equation has two real roots"
   R to mean "the equation has a repeated root"
   N to mean "the equation has no real roots".

   Classify the following equations with T, R or N:

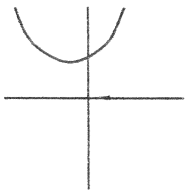
   (a) $x^2 + 9 = 6x$
   (b) $x(x-4) = 200$
   (c) $\dfrac{x}{x-1} = x - 2$

   (d) $(x+2)^2 + 3 = 1$
   (e) $4x^2 + 1 = 4x$
   (f) $4x = x^2 + 5$

5. (a) The equation $x^2 + 8x + k = 0$ has a repeated root. Find the value of k.

   (b) For what values of c will the equation $x^2 - 3x = c$ have two real roots?

   (c) The sketch graph shows $y = x^2 + bx + c$. Write down an inequality connecting b and c.



6. The graph below shows part of a quadratic function $y = q(x)$.

   (a) Giving your answers correct to 1 decimal place, write down the roots of $q(x) = 1$.

   (b) Write down an integer value of k for which the equation $q(x) = k$ will have no real roots.