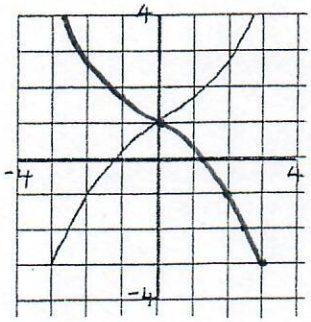
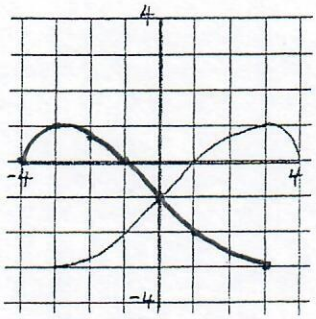
M-10 Transforming Graphs 2 : Reflections, Mixed

1. In each diagram below the given graph is $y = f(x)$. Sketch the transformation of $y = f(x)$ indicated in brackets.

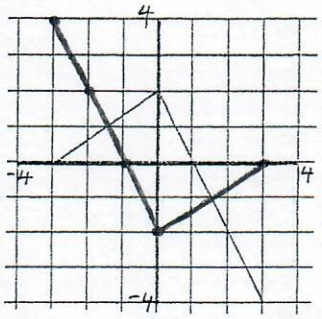
(a) $[y = -f(x)]$



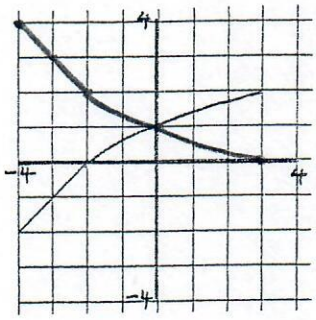
(b) $[y = f(-x)]$



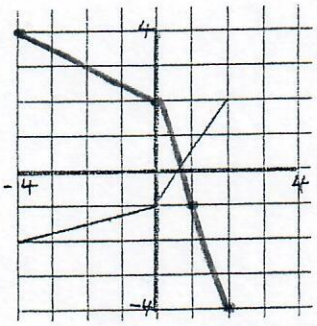
(c) $[y = -f(-x)]$



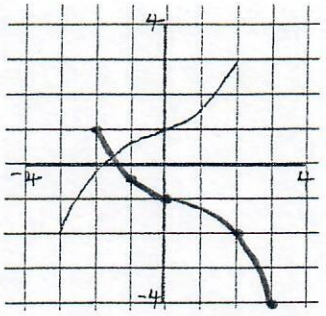
(d) $[y = -f(x) + 2]$



(e) $[y = -2f(x)]$



(f) $[y = f(-x) - 2]$



2. Write down the transformation of $y = f(x)$ represented by the graphs labeled:

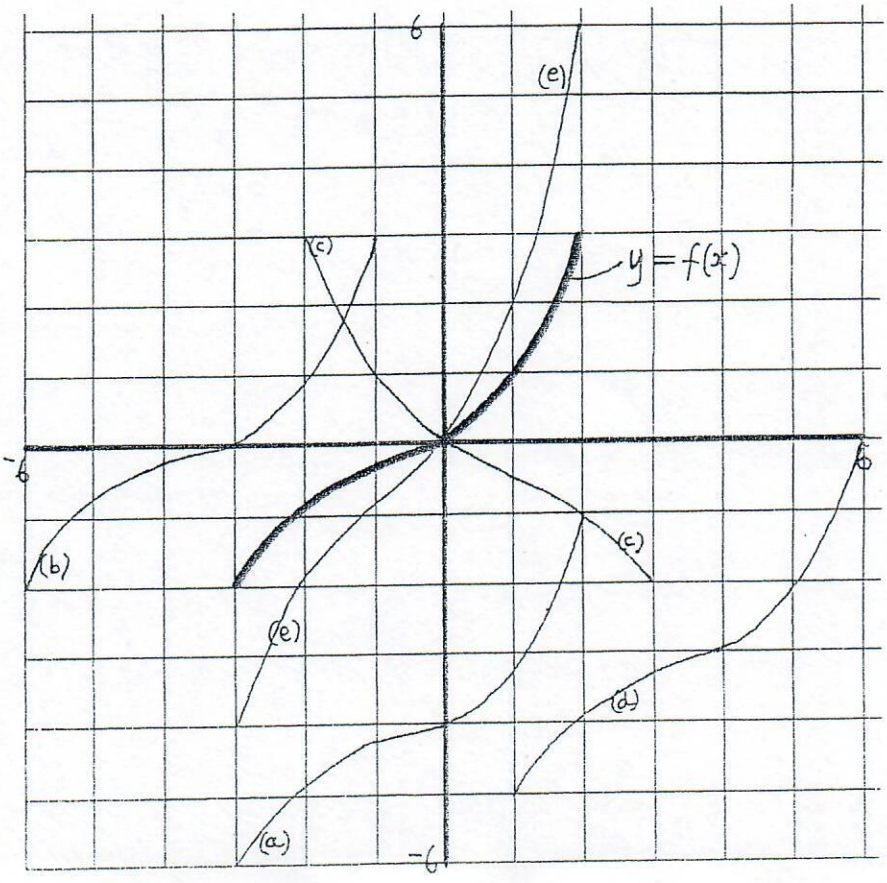
(a) $f(x) - 4$

(b) $f(x+3)$

(c) $f(-x)$

(d) $f(x-4) - 3$

(e) $2f(x)$



M-11 The Quadratic Function

1. Write each quadratic in the form $a(x-h)^2+k$:

(a) $y = x^2+6x+10$ $y=(x+3)^2+1$

(b) $y = x^2-4x-7$ $y=(x-2)^2-11$

(c) $y = x^2+3x$ $y=(x+1.5)^2-2.25$

(d) $y = x^2-8x+10$ $y=(x-4)^2-6$

$y = x^2+4x-12$

(e) $y = (x-2)(x+6)$ $y=(x+2)^2-16$

(f) $y = x^2+5x+1$ $y=(x+2.5)^2-5.25$

Separate page (g) $y = 2x^2+8x+7$

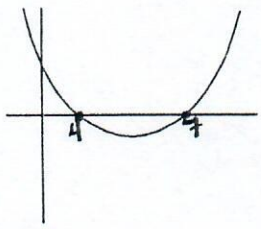
Separate page (h) $y = 1+2x-2x^2$

2. Write down the axis of symmetry of each quadratic in Q1. Separate page

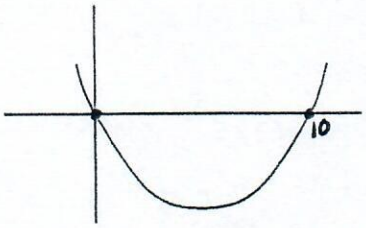
$x=\frac{-b}{2a}$

3. Each sketch graph below represents a quadratic of the form $y = x^2+bx+c$. Find the values of b and c in each case: Separate page

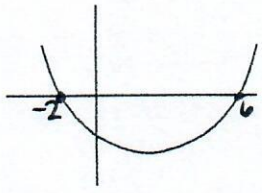
(a)



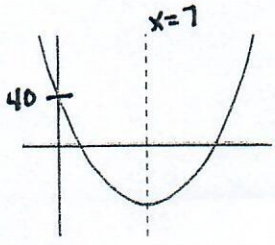
(b)



(c)

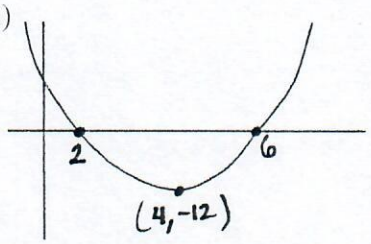


(d)

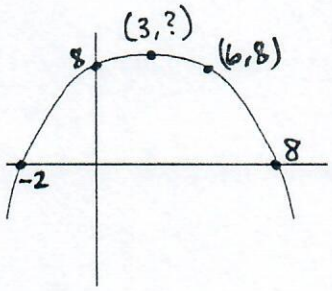


4. Each sketch graph below represents a quadratic of the form $y = ax^2+bx+c$. Find the values of a, b and c in each case: Separate page

(a)



(b)



Sketches may help with these :

5. A quadratic has its vertex at (3, 2) and passes through (5, 14) Separate page

(a) For what other x-value will the y-value = 14?

(b) Find the equation of the quadratic in the form $y = ax^2+bx+c$.

6. A quadratic which passes through (2, 8) has a zero at x = 4. Its y-intercept is 12. Separate page

Find (a) the equation of the quadratic in the form $y = ax^2+bx+c$

(b) the other zero of the quadratic.

7. A quadratic of the form $y = ax^2+bx$ has a line of symmetry at x = 2.5. Find the zeros of the quadratic. Separate page

8. The table for $y = ax^2+bx+c$ is shown. Find the values of a, b and c. Separate page

x	0	1	2
y	-5	0	9

M - 11

1. g) $y = 2x^2 + 8x + 7$

$y = 2(x^2 + 4x + 4) + 7 - 8$

$\boxed{y = 2(x+2)^2 - 1}$

h) $y = -2x^2 + 2x + 1$

$y = -2(x^2 - x + 1/4) + 1 + 1/2$

$\boxed{y = -2(x - 1/2)^2 + 3/2}$

2. a) $x = -3$

b) $x = 2$

c) $x = -1.5$

d) $x = 4$

e) $x = -2$

f) $x = -2.5$

g) $x = -2$

h) $x = 1/2$

3. a) $x = 4, x = 7$

$x - 4 = 0, \quad x - 7 = 0$

$y = (x-4)(x-7)$

$y = x^2 - 11x + 28 \rightarrow \boxed{b = -11, \ c = 28}$

b) $x = 0, \ x = 10$

$x - 0 = 0, \ x - 10 = 0$

$y = x(x-10)$

$y = x^2 - 10x \rightarrow \boxed{b = -10, \ c = 0}$

c) $x = -2, \ x = 6$

$x + 2 = 0, \ x - 6 = 0$

$y = (x+2)(x-6)$

$y = x^2 - 4x - 12 \rightarrow \boxed{b = -4, \ c = -12}$

d) $y = 0^2 + b(0) + c = 40 \rightarrow \boxed{c = 40}$

$y = x^2 + bx + 40$

$7 = \frac{-b}{2a} \rightarrow 7 = \frac{-b}{2(1)} \rightarrow -b = 14 \rightarrow \boxed{b = -14}$

M-11

4. a) $x=2,\ x=6$

$x-2=0,\ x-6=0$

$y=k(x-2)(x-6)$

$y=k(x^2-8x+12)$

$-12=k(4^2-8(4)+12)$

$-12=k(16-32+12)$

$-12=k\cdot(-4)$

$k=3$

$y=3(x^2-8x+12)=3x^2-24x+36 \rightarrow \boxed{a=3,\ b=-24,\ c=36}$

b) $x=-2,\ x=8$

$x+2=0,\ x-8=0$

$y=k(x+2)(x-8)$

$y=k(x^2-6x-16)$

$8=k(0^2-6(0)-16)$

$8=k(0-0-16)$

$8=k(-16)$

$k=-1/2$

$y=-1/2(x^2-6x-16)=-\frac{1}{2}x^2+3x+8 \rightarrow \boxed{a=-1/2,\ b=3,\ c=8}$

5. a) $\boxed{(1,14)}$ point, $(3,2)$ vertex, $(5,14)$ point

$2\leftarrow, 12\uparrow$ $\quad 2\rightarrow, 12\uparrow$

b) $2^2=4$, so $12\uparrow$ is a $3\times$ stretch in height, so $\boxed{a=3}$

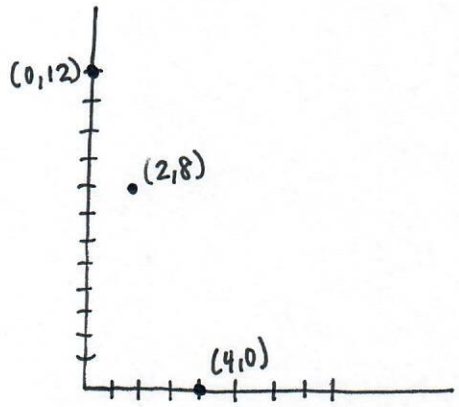
vertex at $x=\frac{-b}{2a}$, so $3=\frac{-b}{2(3)} \rightarrow 3=\frac{-b}{6} \rightarrow -b=18 \rightarrow \boxed{b=-18}$

$y=3x^2-18x+c$

$2=3\cdot3^2-18\cdot3+c \rightarrow 2=27-54+c \rightarrow \boxed{c=29}$

M-11

6. a)



$y = ax^2 + bx + c$

$12 = a\cdot 0^2 + b\cdot 0 + c \rightarrow \boxed{c = 12}$

$y = ax^2 + bx + 12$

$0 = a\cdot 4^2 + b\cdot 4 + 12$ $\qquad$ $8 = a\cdot 2^2 + b\cdot 2 + 12$

$16a + 4b = -12$ $\qquad$ $4a + 2b = -4$

$4a + b = -3$ $\qquad$ $2a + b = -2$

$b = -4a - 3 \longleftrightarrow b = -2a - 2$

$-4a - 3 = -2a - 2$

$-1 = 2a$

$\boxed{a = -1/2}$ $\qquad$ $b = -4(-1/2) - 3$

$b = 2 - 3 \rightarrow \boxed{b = -1}$

b) $y = -\frac{1}{2}x^2 - x + 12$

$-\frac{1}{2}x^2 - x + 12 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(-\frac{1}{2})(12)}}{2(-\frac{1}{2})} = \frac{1 \pm \sqrt{1+24}}{-1} = \frac{1 \pm \sqrt{25}}{-1} = \frac{1 \pm 5}{-1}$

$\frac{6}{-1} = \boxed{-6}$

$\frac{-4}{-1} = 4$ ✓

7. $y = ax^2 + bx$

$\frac{-b}{2a} = 2.5 \rightarrow -b = 5a \rightarrow b = -5a$

$ax^2 + bx = 0 \rightarrow x(ax + b) = 0 \rightarrow \boxed{x = 0}$ or $ax + b = 0$

$x = \frac{-b}{a} = \frac{5a}{a} \rightarrow \boxed{x = 5}$

8. $y = ax^2 + bx + c$

$-5 = a\cdot 0^2 + b\cdot 0 + c \rightarrow \boxed{c = -5}$

$y = ax^2 + bx - 5$

$0 = a\cdot 1^2 + b\cdot 1 - 5$ $\qquad$ $9 = a\cdot 2^2 + b\cdot 2 - 5$

$0 = a + b - 5$ $\qquad$ $9 = 4a + 2b - 5$

$a + b = 5$ $\qquad$ $4a + 2b = 14$

$b = 5 - a$ $\qquad$ $2a + b = 7 \rightarrow 2a + 5 - a = 7$

$a + 5 = 7$ $\qquad$ $b = 5 - a$

$\boxed{a = 2}$ $\qquad$ $b = 5 - 2$

$\boxed{b = 3}$

M-12 Quadratic Equations

1. Solve these quadratic equations using a factorisation method:

(a) $x^2+4x-32=0$ $(x+8)(x-4)=0 \rightarrow x=-8, 4$

(b) $x^2-3x-18=0$ $(x-6)(x+3)=0 \rightarrow x=6, -3$

(c) $x^2+x-42=0$ $(x+7)(x-6)=0 \rightarrow x=-7, 6$

d-l Separate page

(d) $2x^2+x-6=0$

(e) $x^2=5x$

(f) $x^2+x=30$

(g) $3x^2=2x+40$

(h) $\dfrac{x}{x-4}=x-3$

(i) $2x(x-3)=20$

(j) $(x+2)^2=8x+1$

(k) $41=(x+1)^2+(x+2)^2$

(l) $\dfrac{60}{x-1}-2=x+4$

2. Where possible, find the zeros **correct to two decimal places**, using the quadratic formula or otherwise: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

(a) $x^2-2x-20=0$ $5.58, -3.58$

(b) $2x^2-3x-11=0$ $3.21, -1.71$

(c) $x^2+x=1000$
$x^2+x-1000=0$ $31.13, -32.13$

(d) $\dfrac{20}{x-3}=x+1$ $5.90, -3.90$

(e) $x^2=6x-10$
$x^2-6x+10=0$ $3+i, 3-i$

(f) $(x-4)^2=7x$ $13.84, 1.16$

3. (a) Write down a quadratic equation which has roots $x=5$ and $x=-7$. Separate page

(b) Write down a quadratic equation which has a repeated root $x=3$.

(c) Write down (in the form $ax^2+bx+c=0$ with $a, b, c \in \mathbb{Z}$) a quadratic equation which has roots $x=\dfrac{1}{3}$ and $x=-\dfrac{1}{2}$.

(d) $x^2+bx+c=0$ has a repeated root $x=-7$. Find the values of b and c.

Separate page

4. In this question we use:
T to mean "the equation has two real roots"
R to mean "the equation has a repeated root"
N to mean "the equation has no real roots".

Classify the following equations with T, R or N:

(a) $x^2+9=6x$

(b) $x(x-4)=200$

(c) $\dfrac{x}{x-1}=x-2$

(d) $(x+2)^2+3=1$

(e) $4x^2+1=4x$

(f) $4x=x^2+5$

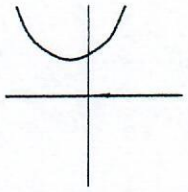
5. (a) The equation $x^2+8x+k=0$ has a repeated root. Find the value of k.

$8^2-4(1)(k)=0$
$64-4k=0$
$\boxed{k=16}$

(b) For what values of c will the equation $x^2-3x=c$ have two real roots?

$(-3)^2-4(1)(-c)>0$
$9+4c>0$
$\boxed{c>-\frac{9}{4}}$

(c) The sketch graph shows $y=x^2+bx+c$. Write down an inequality connecting b and c.



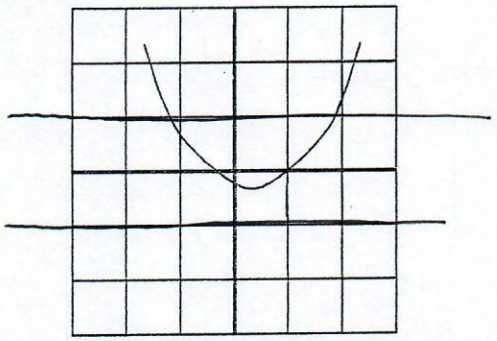
$x^2-3x-c=0$

$b^2-4(1)(c)<0$
$\boxed{b^2-4c<0}$

6. The graph below shows part of a quadratic function $y=q(x)$.

(a) Giving your answers correct to 1 decimal place, write down the roots of $q(x)=1$. $x=-1.2, 1.8$

(b) Write down an integer value of k for which the equation $q(x)=k$ will have no real roots. $k=-1, -2, -3$, etc.



M-12

1. d) $2x^2+x-6=0$

x to -12, $+$ to 1 : $4, -3$

$2x^2+4x-3x-6=0$

$2x(x+2)-3(x+2)=0$

$(x+2)(2x-3)=0 \rightarrow \boxed{x=-2,\ x=3/2}$

e) $x^2=5x$

$x^2-5x=0$

$x(x-5)=0 \rightarrow \boxed{x=0,\ x=5}$

f) $x^2+x=30$

$x^2+x-30=0$

$(x+6)(x-5)=0 \rightarrow \boxed{x=-6,\ x=5}$

g) $3x^2=2x+40$

$3x^2-2x-40=0$

x to -120, $+$ to -2 : $-12, 10$

$3x^2-12x+10x-40=0$

$3x(x-4)+10(x-4)=0$

$(x-4)(3x+10)=0 \rightarrow \boxed{x=4,\ x=-10/3}$

h) $\frac{x}{x-4}=x-3$

$x=(x-3)(x-4)$

$x=x^2-7x+12$

$0=x^2-8x+12$

$0=(x-6)(x-2) \rightarrow \boxed{x=6,\ x=2}$

i) $2x(x-3)=20$

$2x^2-6x-20=0$

$x^2-3x-10=0$

$(x-5)(x+2)=0 \rightarrow \boxed{x=5,\ x=-2}$

M-12

1. j) $(x+2)^2 = 8x+1$

$x^2+4x+4 = 8x+1$

$x^2-4x+3 = 0$

$(x-3)(x-1) = 0 \rightarrow \boxed{x=3, x=1}$

k) $41 = (x+1)^2 + (x+2)^2$

$41 = x^2+2x+1 + x^2+4x+4$

$0 = 2x^2+6x-36$

$0 = x^2+3x-18$

$0 = (x+6)(x-3) \rightarrow \boxed{x=-6, x=3}$

l) $\frac{60}{x-1} - 2 = x+4$

$\frac{60}{x-1} = x+6$

$60 = (x+6)(x-1)$

$60 = x^2+5x-6$

$0 = x^2+5x-66$

$0 = (x+11)(x-6) \rightarrow \boxed{x=-11, x=6}$

2. d) $\frac{20}{x-3} = x+1$

$20 = (x+1)(x-3)$

$20 = x^2-2x-3$

$0 = x^2-2x-23$

e) $x^2-6x+10 = 0$

$x = \frac{6 \pm \sqrt{36-4(1)(10)}}{2(1)} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = \boxed{3 \pm i}$

f) $(x-4)^2 = 7x$

$x^2-8x+16 = 7x$

$x^2-15x+16 = 0$

M-12

3. a) $x=5,\ x=-7$

$x-5=0,\ x+7=0$

$y=(x-5)(x+7)$

$\boxed{y=x^2+2x-35}$

b) $x=3,\ x=3$

$x-3=0,\ x-3=0$

$y=(x-3)(x-3)$

$\boxed{y=x^2-6x+9}$

c) $x=1/3,\ x=-1/2$

$x-1/3=0,\ x+1/2=0$

$y=(x-1/3)(x+1/2)$

$y=x^2+\frac{1}{2}x-\frac{1}{3}x-\frac{1}{6}=x^2+\frac{3}{6}x-\frac{2}{6}x-\frac{1}{6}=x^2+\frac{1}{6}x-\frac{1}{6}$

$6\left(x^2+\frac{1}{6}x-\frac{1}{6}\right)=0\cdot 6$

$\boxed{6x^2+x-1=0}$

d) $x=-7,\ x=-7$

$x+7=0,\ x+7=0$

$y=(x+7)(x+7)$

$y=x^2+14x+49 \rightarrow \boxed{b=14,\ c=49}$

4. a) $x^2-6x+9=0$

$d=(-6)^2-4(1)(9)=36-36=0 \rightarrow \boxed{R}$

b) $x(x-4)=200$

$x^2-4x-200=0$

$d=(-4)^2-4(1)(-200)=16+800=816 \rightarrow \boxed{T}$

c) $\frac{x}{x-1}=x-2$

$x=x^2-3x+2$

$0=x^2-4x+2$

$d=(-4)^2-4(1)(2)=16-8=8 \rightarrow \boxed{T}$

d) $(x+2)^2+3=1$

$x^2+4x+4+3-1=0$

$x^2+4x+6=0$

$d=4^2-4(1)(6)=16-24=-8 \rightarrow \boxed{N}$

e) $4x^2-4x+1=0$

$d=(-4)^2-4(4)(1)$

$d=16-16=0 \rightarrow \boxed{R}$

f) $x^2-4x+5=0$

$d=(-4)^2-4(1)(5)$

$d=16-20=-4 \rightarrow \boxed{N}$