Name Key

M-07 **Functions 1 : Basics**

Answer on this sheet

1. The following functions are given:

$f(x) = 3x - 2$	$g(x) = x^2 - 4$
$h(x) = \sqrt{2x-3}$	$k(x) = \dfrac{10-x}{3}$

(a) Find

(i) $f(4) =$ 10 (ii) $g(6) =$ 32

(iii) $h(26) =$ ± 7 (iv) $k(-2) =$ 4

(v) $f(-\frac{1}{6}) =$ -2.5 (vi) $g(-3.5) =$ 8.25

(vii) $k(0.1) =$ 3.3 (viii) $g(3) - f(3) =$ $5 - 7 = -2$

(ix) $h(1) =$ $\sqrt{-1} = \pm i$ (x) $k(-3.2)$ $= 4.4$

(b) Find the value of x if:

(i) $k(x) = 7$

$\frac{10-x}{3} = 7$

$10 - x = 21$

$x = -11$

(ii) $h(x) = 4$

$\sqrt{2x-3} = 4$

$2x - 3 = 16$

$x = 9.5$

(iii) $f(x) = x$

$3x - 2 = x$

$2x = 2$

$x = 1$

(iv) $k(x) = f(x)$

$\frac{10-x}{3} = 3x - 2$

$10 - x = 9x - 6$

$16 = 10x$

$1.6 = x$

(v) $g(x) \times h(x) = 0$

Either $g(x) = 0$ or $h(x) = 0$

$x^2 - 4 = 0$ $\sqrt{2x-3} = 0$

$x^2 = 4$ $2x - 3 = 0$

$x = \pm 2$ $x = 1.5$

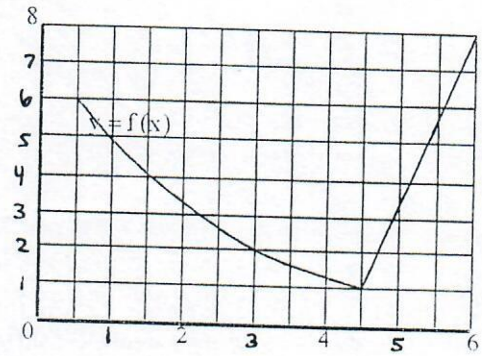
2. Using the functions from Q1, find

(a) $k(k(4)) =$ $k(2) = 8/3$

(b) $h(k(-8)) =$ $h(6) = \pm 3$

(c) $f(f(f(1))) =$ $f(f(1)) = f(1) = 1$

3.



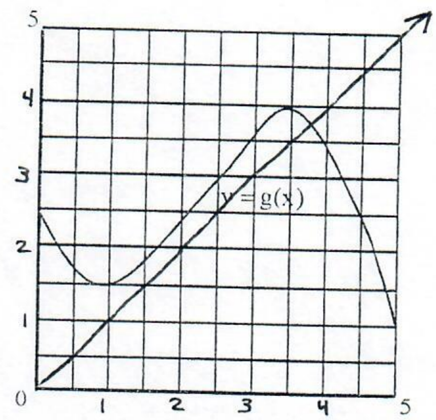
(a) Write down the domain and range of the function graphed above.

D: [0.5, 6] R: [1, 8]

(b) Find $f(3) =$ 2

(c) Solve $f(x) = 4$ $x =$ 1.6, 5.2

4.



(a) Solve $g(x) = 3$ $x =$ 2.5, 4.2

(b) By drawing a line on the graph solve
$g(x) = x$

$x =$ 3.8

(c) Write down a positive integer k such that the equation $g(x) = k$ has three solutions in the given domain.

$k =$ 2

5. (a) Find the range of $g(x) = 10 - 3x$ for the domain $0 \le x \le 3$. [1, 10]

$g(0) = 10$

$g(3) = 1$

(b) Find the range of $h(x) = x^2 - 4x + 6$ for the domain $0 \le x \le 3$. [2, 6]

$h(0) = 6$

M-08 Functions 2 : Composite and Inverse

1. Given these functions:

$$f : x \to 3x - 2$$
$$g : x \to \frac{x}{x-2}$$
$$h : x \to x(6-x)$$

find the values of the following:

(a) $f \circ g(3)$
$f(3) = 7$

(b) $g \circ h(5)$
$g(5) = 5/3$

(c) $f \circ h(2)$
$f(8) = 22$

(d) $f \circ f(5)$
$f(13) = 37$

(e) $g \circ g(-4)$
$g(2/3) = -1/2$

(f) $h \circ g \circ f(2)$
$h(g(4)) = h(2) = 8$

(g) $h \circ g(1)$
$h(-1) = -7$

(h) $g \circ h(1)$
$g(5) = 5/3$

separate page

2. Find the inverse function in each case:

(a) $f(x) = 4x - 1$

(b) $f(x) = 3(x-2)$

(c) $f(x) = \frac{x+2}{5}$

(d) $f(x) = 2(1-2x)$

(e) $f(x) = \frac{10-2x}{3}$

(f) $f(x) = \sqrt{x-5} \quad (x \geq 5)$

(g) $f(x) = 1 + x^3$

separate page

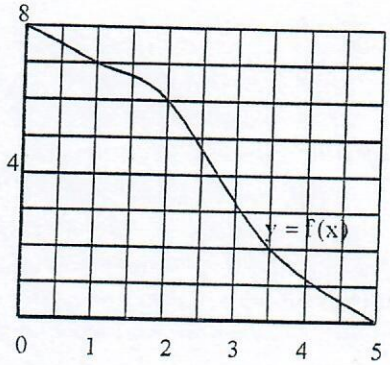
3. $h(x) = \frac{4-x}{3}$

(a) Find $h^{-1}(x)$.

(b) Use the answer to (a) to solve $h^{-1}(x) = -2$.

(c) Find another (simpler!) way to solve $h^{-1}(x) = -2$.

4.



(a) Find (i) $f(2) =$ 6 (ii) $f^{-1}(2) =$ 3.5

(b) Find (i) $f^{-1}(7) =$ 1 (ii) $f(f^{-1}(7)) =$ $f(1) = 7$

(c) Solve $f^{-1}(x) = 3$ $x =$ 3.2

(d) Find $f \circ f(2.5)$ as accurately as you can:

$f \circ f(2.5) =$ $f(4.7) = 0.5$

5. (a) Given $f(x) = 2x + 1$ and $g(x) = 5x$, find $f \circ g(x)$. $2(5x) + 1 = 10x + 1$

(b) Given $f(x) = 1 - 3x$ and $g(x) = 2x + 7$, find $g \circ f(x)$.
$2(1-3x) + 7$
$2 - 6x + 7 = -6x + 9$

(c) Given $f(x) = \frac{x}{2} + 1$ and $g(x) = 6 - 2x$, find $f \circ g(x)$.
$\frac{6-2x}{2} + 1$
$3 - x + 1 = -x + 4$

6. Given the functions

$$f(x) = 3x - 2 \qquad g(x) = \frac{x+2}{5}$$

find:

$x = \frac{y+2}{5}$
$5x = y + 2 \to y = 5x - 2$

(a) $f \circ g(x)$, giving your answer in the form $f \circ g(x) = \frac{ax+b}{c}$ (with $a, b, c \in \mathbb{Z}$)

$3\left(\frac{x+2}{5}\right) - 2$
$\frac{3x+6}{5} - \frac{10}{5} = \frac{3x-4}{5}$

(b) $f \circ (g^{-1})(x)$

$3(5x-2) - 2 = 15x - 6 - 2 = 15x - 8$

2. a) $y = 4x - 1$

$x = 4y - 1$

$x + 1 = 4y$

$\boxed{y = \frac{x+1}{4}}$

b) $y = 3(x-2)$

$y = 3x - 6$

$x = 3y - 6$

$x + 6 = 3y$

$\boxed{y = \frac{x+6}{3}}$

c) $y = \frac{x+2}{5}$

$x = \frac{y+2}{5}$

$5x = y + 2$

$\boxed{y = 5x - 2}$

d) $y = 2(1-2x)$

$y = 2 - 4x$

$x = 2 - 4y$

$x - 2 = -4y$

$\boxed{y = \frac{x-2}{-4} = -\frac{1}{4}x + \frac{1}{2}}$

e) $y = \frac{10-2x}{3}$

$x = \frac{10-2y}{3}$

$3x = 10 - 2y$

$2y = -3x + 10$

$\boxed{y = -\frac{3}{2}x + 5}$

M - 8

f) $y = \sqrt{x-5}$

$x = \sqrt{y-5}$

$x^2 = y - 5$

$\boxed{y = x^2 + 5}$

g) $y = 1 + x^3$

$x = 1 + y^3$

$x - 1 = y^3$

$\boxed{y = \sqrt[3]{x-1}}$

3. $h(x) = \frac{4-x}{3}$

a) $y = \frac{4-x}{3}$

$x = \frac{4-y}{3}$

$3x = 4 - y$

$\boxed{y = -3x + 4}$

b) $-3x + 4 = -2$

$-3x = -6$

$\boxed{x = 2}$

c) $h^{-1}(x) = -2$

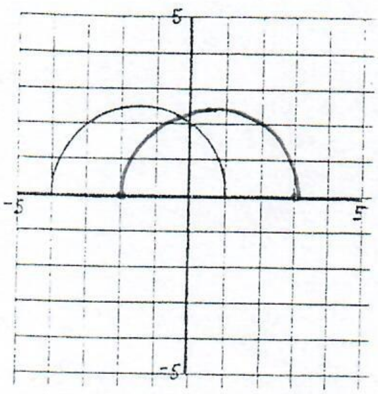
y value of $h^{-1}(x)$ = x value of $h(x)$

$h(-2) = \frac{4-(-2)}{3} = \frac{6}{3} = \boxed{2}$ ✓

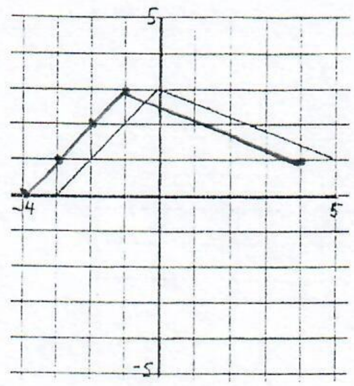
M-09 **Transforming Graphs 1 : Translations, Stretches**

In each diagram the below the given graph is $y = f(x)$. Sketch the transformation of $y = f(x)$ indicated in brackets.

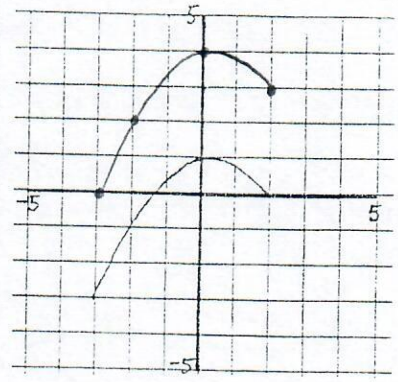
(a) $[y = f(x-2)]$



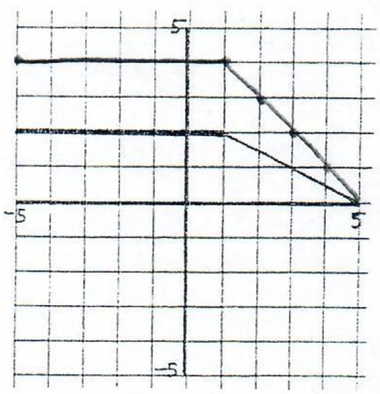
(b) $[y = f(x+1)]$



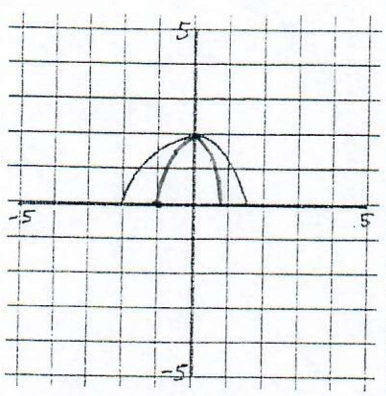
(c) $[y = f(x)+3]$



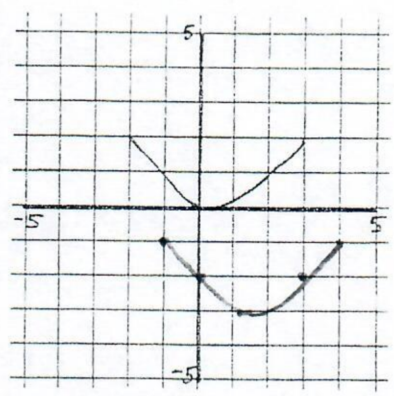
(d) $[y = 2f(x)]$



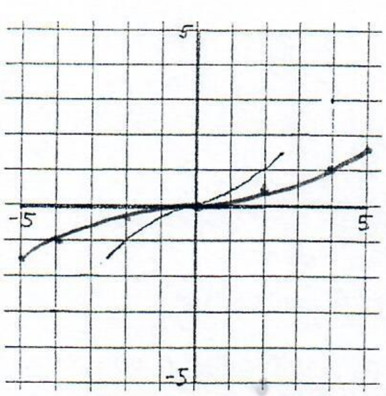
(e) $[y = f(2x)]$



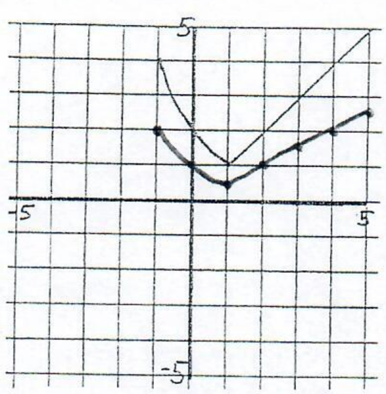
(f) $[y = f(x-1)-3]$



(g) $\left[y = f\left(\dfrac{x}{2}\right)\right]$



(h) $\left[y = \dfrac{f(x)}{2}\right]$



(i) $[y = f(2x)-3]$