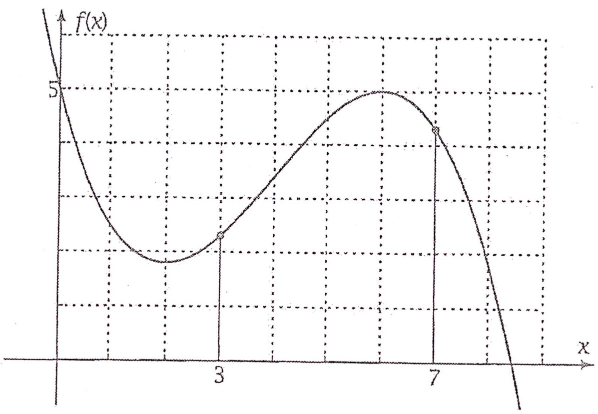
Name: ______________ Group Members: ______________

Exploration 5-5a: The Mean Value Theorem

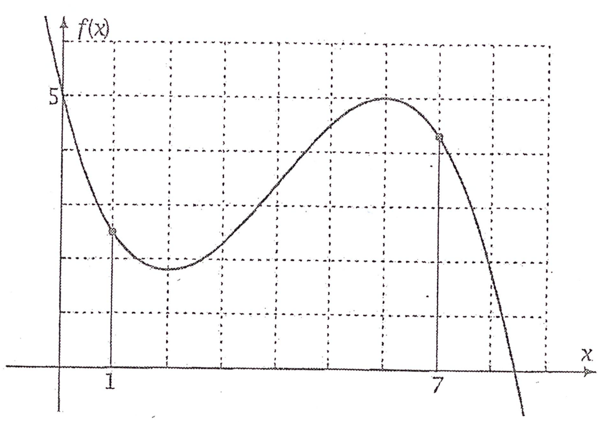
Date: ______________

Objective: Without looking at the text, discover the hypotheses and conclusion of the mean value theorem.

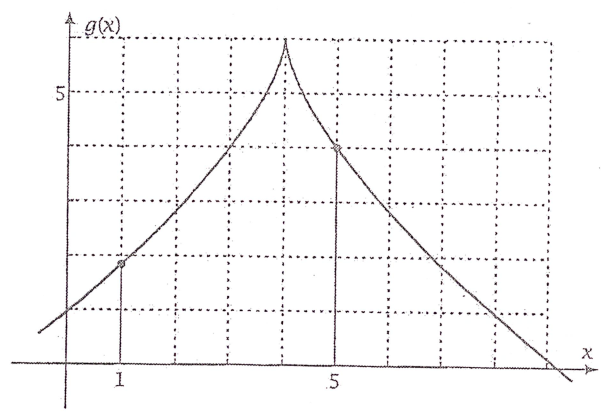
1. For $f(x) = -0.1x^3 + 1.2x^2 - 3.6x + 5$, graphed below, there is a value of $x = c$ between 3 and 7 at which the tangent to the graph is parallel to the secant line through $(3, f(3))$ and $(7, f(7))$.
   - Draw the secant line and the tangent line.
   - From the graph, $c \approx$ ______
   - Is f differentiable on $(3, 7)$? ______
   - Is f continuous on $[3, 7]$? ______



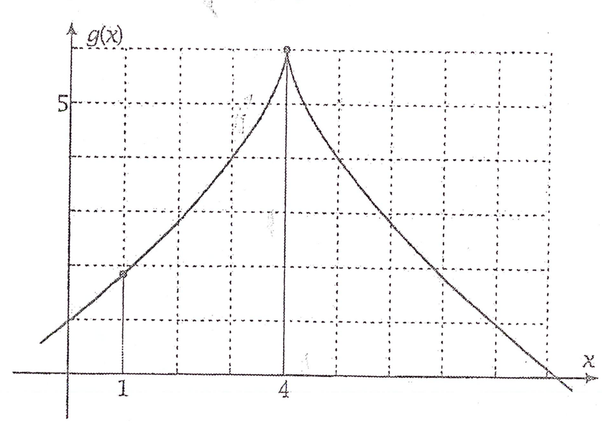
2. Function f from Problem 1 has *two* values of $x = c$ between $x = 1$ and $x = 7$ at which $f'(c)$ equals the slope of the corresponding secant line. (That is, the tangent line parallels the secant line.)
   - Draw the secant and tangents on the graph below.
   - From the graph, $c \approx$ ______ and $c \approx$ ______
   - Is f differentiable on $(1, 7)$? ______
   - Is f continuous on $[1, 7]$? ______



3. For $g(x) = 6 - 2(x - 4)^{2/3}$, graphed below,
   - Draw a secant line through $(1, g(1))$ and $(5, g(5))$.
   - Is g differentiable on $(1, 5)$? ______
   - Is g continuous on $[1, 5]$? ______
   - Tell why there is *no* value of $x = c$ between $x = 1$ and $x = 5$ at which $g'(c)$ equals the slope of the secant line.



4. Function g from Problem 3 *does* have a value $x = c$ in $(1, 4)$ for which $g'(c)$ equals the slope of the secant line through $(1, g(1))$ and $(4, g(4))$.
   - Draw the secant line and tangent line, below.
   - From the graph, $c \approx$ ______
   - Is g differentiable on $(1, 4)$? ______
   - Is g continuous on $[1, 4]$? ______



(Over)

Name: ____________ Group Members: ____________

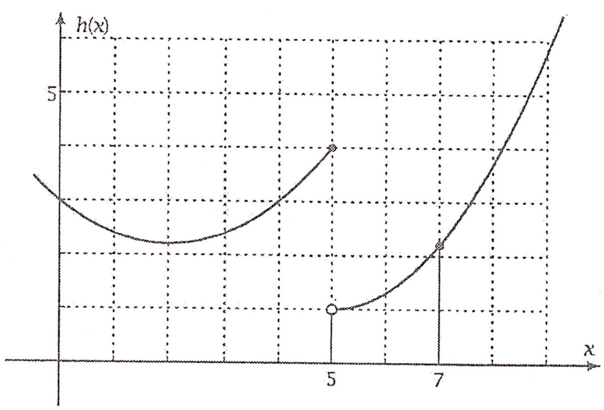
Exploration 5-5a: The Mean Value Theorem *continued*

Date: ____________

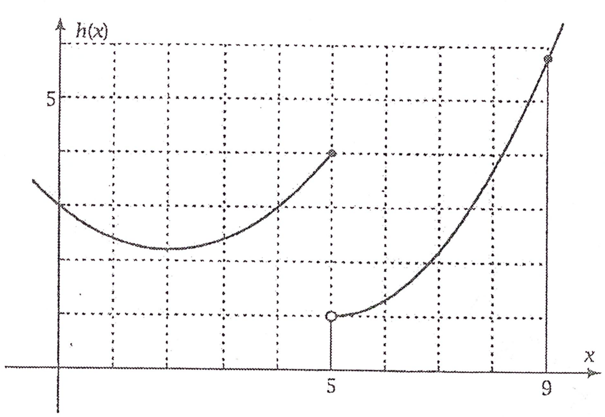
5. Piecewise function h is defined by

$$h(x) = \begin{cases} 0.2(x-2)^2 + 2.2, & \text{if } x \le 5 \\ 0.3(x-5)^2 + 1, & \text{if } x > 5 \end{cases}$$

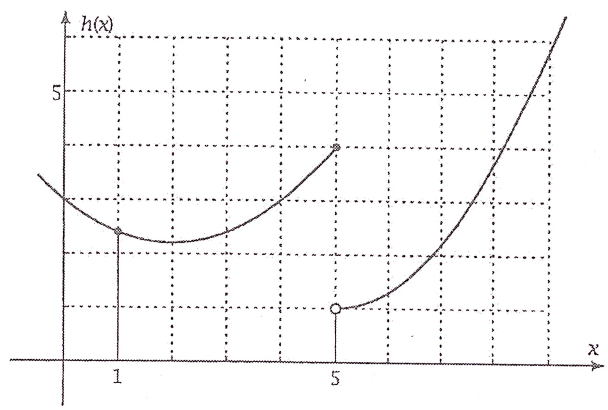
- Draw a secant line through $(5, h(5))$ and $(7, h(7))$.
- Is h differentiable on $(5, 7)$? ________
- Is h continuous on $[5, 7]$? ________
- Why is there *no* value $x = c$ in $(5, 7)$ for which $h'(c)$ equals the slope of the secant line?



6. The graph below is function h from Problem 5.
- Draw a secant line through $(5, h(5))$ and $(9, h(9))$.
- Is h differentiable on $(5, 9)$? ________
- Is h continuous on $[5, 9]$? ________
- There *is* a point $x = c$ in $(5, 9)$ where $h'(c)$ equals the slope of the secant line. Draw the tangent line. Estimate the value of c. ________



7. The graph below is function h from Problem 5.
- Draw a secant line through $(1, h(1))$ and $(5, h(5))$.
- Show that there is a point $x = c$ in $(1, 5)$ where $h'(c)$ equals the slope of the secant line.
- Is h differentiable on $(1, 5)$? ________
- Explain why h is continuous on $[1, 5]$, even though there is a step discontinuity at $x = 5$.



8. The mean value theorem states:
If f is differentiable on (a, b) and
f is continuous on $[a, b]$,
then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{i.e., the secant's slope}$$

For which problem(s) are
- the hypotheses and conclusion true? ________
- the hypotheses and conclusion not true? ________
- the conclusion true, but not the hypotheses? ________

9. The number $x = c$ in the above problems is the "mean" value referred to in the name "mean value theorem." Explain why the hypotheses are sufficient conditions for the conclusion, but *not* necessary conditions.

10. What did you learn as a result of doing this Exploration that you did not know before?