Motion Packet

* Things to remember: speed $= \sqrt{[x'(t)]^2+[y'(t)]^2}$

distance $= \int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$

position: starting point + distance traveled

$\downarrow$

given + $\int x'dt$ or $\int y'dt$

2010

a) $x(t) = t^2-4t+8$

$y_1 \rightarrow x'(t) = 2t-4$

$y_2 \rightarrow y'(t) = te^{t-3}-1$

speed @ $t=3$

$y_3 \rightarrow \text{speed} = \sqrt{[x'(3)]^2+[y'(3)]^2} = \boxed{2.828 \text{ m/sec}}$

$(y_1)^2 + (y_2)^2$ @ $t=3$

b) distance $= \int_0^4 \underbrace{\sqrt{[x'(t)]^2+[y'(t)]^2}}_{y_3}\,dt = \boxed{11.588 \text{ meters}}$

c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ Horz Tang $\rightarrow \frac{dy}{dt}=0$ $\quad \underbrace{te^{t-3}-1}_{y_2} = \underbrace{0}_{y_4}$ intersect y_2 & y_4

$\boxed{t = 2.20794}$

$x'(2.20794) > 0 \therefore$ moving right @ $t = 2.20794$.

d) $x(t) = 5$

$t = 1, 3$

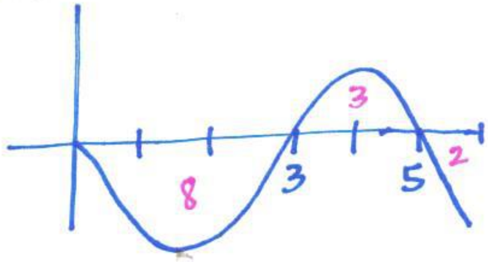
$\frac{dy}{dt}$ @ $t=1 \rightarrow 0.432$

$t=3 \rightarrow 1.000$

$y(1) = y(3) = y(2) + \underbrace{\int_2^3 y'(t)\,dt}_{\text{calc value}}$

$3 + \frac{1}{e} + \nearrow \boxed{= 4.000}$

2008



@ $t=0 \rightarrow x=-2$

a) $x(3) = x(0) + \int_0^3 v(t)\,dt$

$= -2 + -8 = -10$

$x(5) = x(3) + \int_3^5 v(t)\,dt$

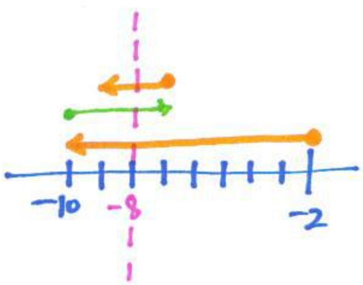
$= -10 + 3 = -7$

$x(6) = x(5) + \int_5^6 v(t)\,dt$

$= -7 + -2 = -9$

$\boxed{\therefore \text{ farthest left occurs @ } t=3 \text{ and is at } -10.}$

b)



$\therefore$ it cross -8 three times.

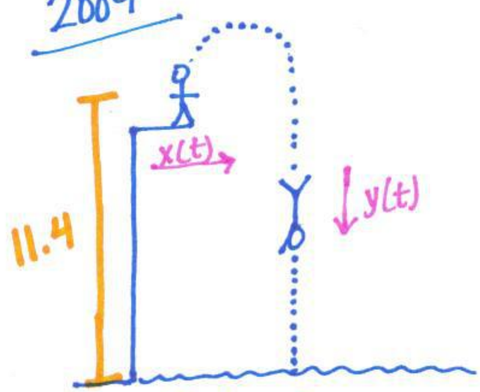
write it in WORDS!

- $[0,3]$ moves left 8 units to -10
- $[3,5]$ moves right 3 units to -7
- $[5,6]$ moves left 2 units to -9.

c) $2 < t < 3 \rightarrow v < 0$ $\therefore$ speed is decreasing

d) accel $< 0 \rightarrow$ slope of vel < 0 $\therefore$ occurs @ $0 < t < 1$ and $4 < t < 6$.

2009



$x'(t) = .8$

$y'(t) = 3.6 - 9.8t$

a) max vert $\rightarrow y' = 0$ $\quad 3.6 - 9.8t = 0$

$t = \underbrace{.36734694}_{b}$

$y(b) = 11.4 + \int_0^b y'(t)\,dt$

$\boxed{= 12.061 \text{ meters}}$

2009 con't

b) $y(A) = 11.4 + \int_0^A (3.6 - 9.8t)\,dt = 0$

$11.4 + 3.6t - 4.9t^2 \Big|_0^A = 0$

$\underbrace{11.4 + 3.6A - 4.9A^2}_{\text{Quad. Formula}} = 0$

$\boxed{A = 1.936 \text{ sec}}$

c) Arc Length: $L = \int_0^A \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt = \boxed{12.946 \text{ meters}}$

d) $\frac{dy}{dx}$ @ time $= A \rightarrow -19.216$

$\boxed{\theta = \tan^{-1}(19.216) = 1.519}$

2008 Form B

a) $x'(t) = \sqrt{3t} \rightarrow y_1$ @ $t = 4 \rightarrow (1,5)$

$y'(t) = 3\cos(t^2/2) \rightarrow y_2$

$a(4) = \langle x''(4), y''(4) \rangle = \boxed{\langle 0.433, -11.872 \rangle}$

b) $y(0) = y(4) + \int_4^0 y'(t)\,dt = 5 + \int_4^0 3\cos(t^2/2)\,dt$

$\boxed{= 1.601}$

c) $\underset{y^3}{\text{speed}} = \sqrt{[\underset{y_1}{x'(t)}]^2 + [\underset{y^2}{y'(t)}]^2} = \underset{y^4}{3.5}$ intersect y_3 & y_4

$\boxed{t = 2.226}$

2008 Form B con't

d) $\int_0^4 \underbrace{\sqrt{[x'(t)]^2+[y'(t)]^2}}_{y_3}\, dt = \boxed{13.182}$

2007 Form B

$\underset{y_1}{x'(t)} = \tan^{-1}\left(\frac{t}{1+t}\right) \quad \underset{y_2}{y'(t)} = \ln(t^2+1) \quad t=0 \rightarrow (-3,-4)$

a) $\underset{y_3}{\text{speed}} = \sqrt{[\underset{y_1}{x'(4)}]^2+[\underset{y_2}{y'(4)}]^2} \boxed{= 2.912}$ @ $t=4$

b) $\text{distance} = \int_0^4 \underbrace{\sqrt{[x'(t)]^2+[y'(t)]^2}}_{y_3}\, dt = \boxed{6.423}$

c) $x(4) = x(0) + \int_0^4 x'(t)\, dt$

$= -3 + 2.10794 = \boxed{-0.892}$

d) $\frac{dy}{dx} = 2 \rightarrow \frac{dy/dt}{dx/dt} = 2$

$\underset{y_1}{\ln(t^2+1)} = \underset{y_2}{2\tan^{-1}(t/1+t)}$ intersect y_1 with $2y_2$.

$t = 1.35766$

accel: $\langle x''(1.35766), y''(1.35766)\rangle$

$\boxed{= \langle 0.135, 0.955\rangle}$

2006

$x'(t) = \sin^{-1}(1-2e^{-t})$ $\quad y'(t) = \dfrac{4t}{1+t^3}$ $\quad t=2 \rightarrow (6,-3)$

y_1 $\quad y_2$

a) accel: $\langle x''(2), y''(2) \rangle = \boxed{\langle 0.396, -0.740 \rangle}$

@ t=2

speed $= \sqrt{[x'(2)]^2 + [y'(2)]^2}$ $\boxed{= 1.208}$

y_1 $\quad y_2$ @ t=2

b) vert tangent $\rightarrow x'(t) = 0$

$\sin^{-1}(1-2e^{-t}) = 0$

$\boxed{t = 0.693}$

c) $m(t) = \dfrac{dy/dt}{dx/dt} = \dfrac{4t/1+t^3}{\sin^{-1}(1-2e^{-t})}$

$\lim\limits_{t\to\infty} m(t) = \lim\limits_{t\to\infty} \underbrace{\dfrac{4t}{1+t^3}}_{\to 0} \cdot \dfrac{1}{\sin^{-1}(1-2e^{-t})} = 0 \cdot \dfrac{1}{\sin^{-1}(1)} \boxed{= 0}$

d) $\lim\limits_{t\to\infty} x(t) = \infty$ so... $C = \lim\limits_{t\to\infty} y(t) \boxed{= y(2) + \int_2^\infty \left(\dfrac{4t}{1+t^3}\right) dt}$

2006 Form B

y_1 $\quad y_2$

$x'(t) = \tan(e^{-t})$ $\quad y'(t) = \sec(e^{-t})$ $\quad$ @ $t=1 \rightarrow (2,-3)$

a) slope: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\sec(e^{-t})}{\tan(e^{-t})}$ @ $t=1 \longrightarrow 2.781$

@ t=1

$\boxed{y+3 = 2.781(x-2)}$

2006 Form B con't

b) accel: $\langle x''(1), y''(1)\rangle = \boxed{\langle -0.423, -0.152\rangle}$

@ t=1

speed $= \sqrt{[x'(1)]^2 + [y'(1)]^2} = \boxed{1.139}$

y_3 y_1 y_2 @ t=1

c) distance: $\int_1^2 \underbrace{\sqrt{[x'(t)]^2+[y'(t)]^2}}_{y_3}\,dt = \boxed{1.059}$

d) $x(0) = x(1) + \int_1^0 x'(t)\,dt$

$2 - 0.77553 > 0$ ∴ object is moving right &
since it starts right of the
y-axis, it is never on the y-axis.

2005 Form B

y_1 y_2

$x'(t) = 12t - 3t^2$ $y'(t) = \ln\left[1+(t-4)^4\right]$ @ $t=0 \to (-13, 5)$

$t=2 \to$ P has x-coor = 3.

a) accel: $\langle x''(2), y''(2)\rangle = \boxed{\langle 0, -1.882\rangle}$

speed $= \sqrt{[x'(2)]^2 + [y'(2)]} = \boxed{12.330}$

y_3 y_1 y_2 @ t=2

y_2

b) $y(2) = y(0) + \int_0^2 y'(t)\,dt = 5 + \int_0^2 \ln\left[1+(t-4)^4\right]dt$

$\boxed{= 13.671}$

2005 Form B con't

c) $\frac{dy}{dx}$ @ $t=2 \rightarrow \frac{dy/dt}{dx/dt} = \frac{\ln(17)}{12} = 0.236$

$$\boxed{y - 13.671 = 0.236\,(x-3)}$$

found in part b) given

d) $x'(t)=0$ $\quad\quad$ $y'(t)=0$

$12t-3t^2=0$ $\quad\quad$ $\ln[1+(t-4)^4]=0$

$t=0,4$ $\quad\quad$ $t=4$ $\quad$ $\therefore$ $t=4$ the object is @ rest because both $x'(t)$ & $y'(t)$ equal zero.

2004

$x'(t) = 3+\cos t^2$ $\quad$ @ $t=2 \rightarrow (1,8)$

y_1

a) $x(4) = x(2) + \int_2^4 x'(t)\,dt$

$= 1 + \int_2^4 (3+\cos t^2)\,dt = \boxed{7.133}$

b) slope @ $t=2 \rightarrow \frac{dy/dt}{dx/dt} = \frac{-7}{3+\cos 4} = \boxed{-2.983}$

if $y'(2)=-7$

c) speed $= \sqrt{[x'(2)]^2+[y'(2)]^2} = \boxed{7.383}$

y_1 $\quad$ $(-7)^2$

d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx}\cdot\frac{dx}{dt} = (2t+1)(3+\cos t^2)$

$y''(4) = 24.814$ $\quad\quad$ $\boxed{\text{accel: } \langle 2.303, 24.814\rangle}$