2005 Form B con't

c) $\frac{dy}{dx}$ @ $t=2 \rightarrow \frac{dy/dt}{dx/dt} = \frac{\ln(17)}{12} = 0.236$

$$\boxed{y - \underset{\substack{\uparrow \\ \text{found in} \\ \text{part b)}}}{13.671} = 0.236\,(x - \underset{\substack{\uparrow \\ \text{given}}}{3})}$$

d) $x'(t) = 0$ $\quad$ $y'(t) = 0$

$12t - 3t^2 = 0$ $\quad$ $\ln[1+(t-4)^4] = 0$

$\downarrow$ $\quad$ $\downarrow$

$t = 0, 4$ $\quad$ $t = 4$ $\quad$ $\therefore$ $t=4$ the object is @ rest because both $x'(t)$ & $y'(t)$ equal zero.

2004

$x'(t) = 3 + \overset{\swarrow y_1}{\cos t^2}$ $\quad$ @ $t = 2 \rightarrow (1, 8)$

a) $x(4) = x(2) + \int_2^4 x'(t)\,dt$

$= 1 + \int_2^4 (3 + \cos t^2)\,dt = \boxed{7.133}$

b) slope @ $t=2 \rightarrow \frac{dy/dt}{dx/dt} = \frac{-7}{3+\cos 4} = \boxed{-2.983}$

if $y'(2) = -7$

c) speed $= \sqrt{[\underset{\substack{\uparrow \\ y_1}}{x'(2)}]^2 + [\underset{\substack{\uparrow \\ (-7)^2}}{y'(2)}]^2} = \boxed{7.383}$

d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t+1)(3+\cos t^2)$

$y''(4) = 24.814$ $\quad$ $\boxed{\text{accel: } \langle 2.303, 24.814 \rangle}$

2004 Form B

$x'(t) = \overset{y_1}{\sqrt{t^4+9}} \qquad y'(t) = \overset{y_2}{2e^{t}+5e^{-t}} \qquad @\ t=0 \rightarrow (4,1)$

a) $\text{speed} = \sqrt{[\underset{y_1}{x'(0)}]^2+[\underset{y_2}{y'(0)}]^2} = \boxed{\sqrt{58} \approx 7.616}$ $\quad y_1 \quad y_2 \ @\ t=0$

accel: $\langle x''(0), y''(0)\rangle = \boxed{\langle 0, -3\rangle}$

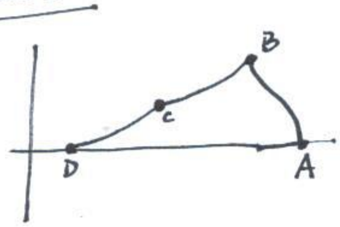
b) slope $\rightarrow$ @ $t=0$ $\dfrac{dy/dt}{dx/dt} = \dfrac{7}{3} \approx 2.\overline{3}$ $\qquad \boxed{y-1 = 2.\overline{3}(x-4)}$

c) $\text{distance} = \int_0^3 \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt \ \boxed{= 45.227}$

d) $x(3) = x(0) + \int_0^3 x'(\overset{y_1}{t})\,dt$

$4 + \int_0^3 (t^4+9)^{1/2}\,dt \ \boxed{= 17.931}$

2003



$x'(t) = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$

$y'(t) = ?$ $\qquad @\ t=9 \rightarrow$ position D.

a) At point C: y-values are decreasing as t increases $\therefore \frac{dy}{dt}$ is <u>not</u> positive.

x-values are decreasing $\therefore \frac{dx}{dt}$ is <u>not</u> positive

2003 con't

b) slope undefined $\rightarrow \frac{dx}{dt}=0$ so either $\cos\left(\frac{\pi t}{6}\right)=0$ or $\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)=0$

$t=3,9,\dots \qquad t=3,15,\dots$

$\boxed{\therefore\ t=3 \text{ for both so point B is @ } t=3.}$

c) $\frac{dy}{dx}=\frac{5}{9}$ @ $t=8 \qquad x'(8)=-9/2$

$\boxed{\text{velocity: } \langle -9/2, -5/2 \rangle}$

$\frac{dy/dt}{dx/dt}=5/9 \longrightarrow \frac{y'(8)}{x'(8)}=\frac{5}{9}$

$y'(8)=\frac{5}{9}\left(-\frac{9}{2}\right)=-\frac{5}{2}$

speed $=\sqrt{(-9/2)^2+(-5/2)^2}\ \boxed{=5.148}$

d) $x(9)-x(0)=\int_0^9 x'(t)\,dt=-39.255$

$\boxed{\therefore\ \text{points A \& D are 39.255 units apart.}}$

2003 Form B (NO CALC)

$x(t)=2e^{3t}+e^{-7t} \qquad y(t)=3e^{3t}-e^{-2t}$

a) vel : $\langle x'(t), y'(t)\rangle = \boxed{\langle 6e^{3t}-7e^{-7t},\ 9e^{3t}+2e^{-2t}\rangle}$

speed : $\sqrt{[x'(0)]^2+[y'(0)]^2}=\sqrt{(6-7)^2+(9+2)^2}$

$=\sqrt{1+121}\ \boxed{=\sqrt{122}}$

b) $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{9e^{3t}+2e^{-2t}}{6e^{3t}-7e^{-7t}}$

$\lim_{t\to\infty}\frac{9e^{3t}+\cancel{2/e^{2t}}^{\,0}}{6e^{3t}-\cancel{7/e^{7t}}^{\,0}}=\frac{9\cancel{e^{3t}}}{6\cancel{e^{3t}}}=\frac{9}{6}=\boxed{\frac{3}{2}}$

2003 Form B con't

c) Horz line → $y'(t)=0$

$9e^{3t}+2e^{-2t} > 0$ for all t, so none exist.

d) Vert line → $x'(t)=0$

$6e^{3t}-7e^{-7t}=0$

$6e^{3t}=7e^{-7t}$

$e^{10t}=7/6$

$\boxed{t=\frac{1}{10}\ln(7/6)}$

2002

$x(t)=10t+4\sin t \qquad y(t)=(20-t)(1-\cos t)$

$y_1 \rightarrow x'(t)=10+4\cos t \qquad y'(t)=(20-t)\sin t+\cos t-1 \leftarrow y_2$

a) slope @ $t=2$: $\dfrac{y'(2)}{x'(2)}=\boxed{1.794}$

b) accel : $\langle x''(t), y''(t)\rangle$

$=\langle x''(13.647083), y''(13.647083)\rangle$

$\boxed{=\langle -3.529, 1.226\rangle}$

$x(t)=10t+4\sin t=140$

↓

$t=13.647083$

c) Max height : $y'(t)=0$

↓

$t=3.024$

speed: $\sqrt{[x'(3.024)]^2+[y'(3.024)]^2}$

$\boxed{=6.028 \text{ m/sec}}$

2002 Con't

d) $y(t)=0$ $\downarrow$ $t=2\pi, 4\pi$

Ave Speed $= \frac{1}{4\pi-2\pi}\int_{2\pi}^{4\pi}\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$

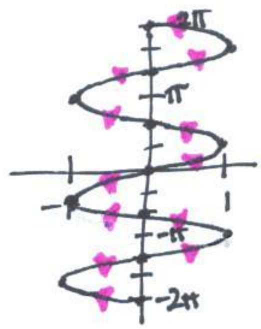
$$\boxed{= \frac{1}{2\pi}\int_{2\pi}^{4\pi}\sqrt{(10+4\cos t)^2+[(20-t)\sin t+\cos t-1]^2}\,dt}$$

2002 Form B

$x(t)=\sin(3t)$ $\quad y(t)=2t$ $\quad -\pi\le t\le\pi$

a)

t	$x=\sin(3t)$	$y=2t$
0	$\sin 0=0$	0
$\pi/6$	$\sin\pi/2=1$	$\pi/3$
$\pi/3$	$\sin\pi=0$	$2\pi/3$
$\pi/2$	$\sin 3\pi/2=-1$	π
$2\pi/3$	$\sin 2\pi=0$	$4\pi/3$
$5\pi/6$	$\sin 5\pi/2=1$	$5\pi/3$
π	$\sin 3\pi=0$	2π



b) $-1\le x(t)\le 1$; $-2\pi\le y(t)\le 2\pi$

c) $x'(t)=0$

$3\cos 3t=0$ $\downarrow$ $\boxed{t=\pi/6}$

Speed: $\sqrt{(3\cos\pi/2)^2+(2)^2}$

$=\sqrt{9(0)+4}$

$\boxed{=2}$

d) distance: $\int_{-\pi}^{\pi}\sqrt{9\cos^2(3t)+4}\,dt$

$=17.973>5\pi$

2001

$$x'(t) = \overset{y_1}{\cos(t^3)} \qquad y'(t) = \overset{y_2}{3\sin(t^2)} \qquad 0 \le t \le 3$$

@ $t=2 \rightarrow (4,5)$

a) slope @ $t=2$: $\dfrac{y'(2)}{x'(2)} = 15.604$ $\boxed{y-5 = 15.604(x-4)}$

b) $\overset{y_3}{\text{speed}}$ @ $t=2$: $\sqrt{[\underset{y_1}{x'(2)}]^2 + [\underset{y_2}{y'(2)}]^2}$ @ $t=2$ $\boxed{= 2.275}$

c) distance $= \displaystyle\int_0^1 \underbrace{\sqrt{[x'(t)]^2 + [y'(t)]^2}}_{y_3}\,dt = \boxed{1.458}$

d) $x(3) = \overset{4}{x(2)} + \displaystyle\int_2^3 x'(t)\,dt \;\boxed{= 3.954}$

$y(3) = \overset{5}{y(2)} + \displaystyle\int_2^3 y'(t)\,dt \;\boxed{= 4.906}$

2000 NO CALC

$$x'(t) = 1 - t^{-2} \qquad y'(t) = 2 + t^{-2} \qquad @\ t=1 \rightarrow (2,6)$$

a) accel: $\langle x''(3), y''(3) \rangle$ $\qquad x''(t) = 2t^{-3} \qquad y''(t) = -2t^{-3}$

$\boxed{\langle 2/27, -2/27 \rangle}$ $\qquad x''(3) = 2/27 \qquad y''(3) = -2/27$

b) position: $x(t) = \int (1 - t^{-2})\,dt$ $\qquad y(t) = \int (2 + t^{-2})\,dt$

$= t + t^{-1} + C_1$ $\qquad = 2t - t^{-1} + C_2$

$x(1) = 1 + 1 + C_1 = 2$ $\qquad y(1) = 2 - 1 + C_2 = 6$

$C_1 = 0$ $\qquad C_2 = 5$

2000 con't

b) $x(t) = t + t^{-1} + 0 \qquad y(t) = 2t - t^{-1} + 5$

$x(3) = 3 + \frac{1}{3} \qquad y(3) = 6 - 1/3 + 5$

$= 10/3 \qquad = 32/3$

$\boxed{\therefore \text{ position @ } t=3: \ (10/3, 32/3)}$

c) $\frac{dy}{dx} = 8 \qquad \frac{dy/dt}{dx/dt} = 8$

$2 + t^{-2} = 8(1 - t^{-2})$

$2 + t^{-2} = 8 - 8t^{-2}$

$9t^{-2} = 6$

$t^{-2} = 6/9$

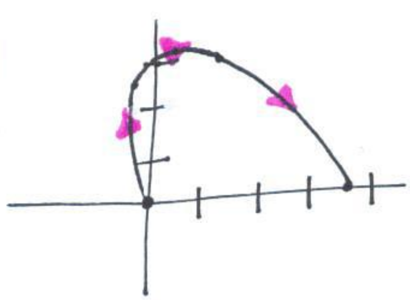
$\boxed{t = \sqrt{9/6} = \sqrt{3/2}}$

d) $\lim_{t \to \infty} dy/dx = \lim_{t \to \infty} \frac{2 + \cancel{1/t^2}^{\to 0}}{1 - \cancel{1/t^2}^{\to 0}} = \boxed{2}$

1999

$x(t) = \frac{1}{2}t^2 - \ln(1+t) \qquad y(t) = 3\sin t \qquad 0 \le t \le \pi$

a)



b) $x'(t) = t - \frac{1}{1+t} = 0$

$\underbrace{t^2 + t - 1 = 0}_{\text{Quad. Form}} \quad \boxed{t = 0.618}$

$x(.618) = \boxed{-0.290}$

$y(.618) = \boxed{1.738}$

1999 con't

c) $x(t) = 0$
$\downarrow$
$t = 1.286$

speed: $\sqrt{[x'(1.286)]^2 + [y'(1.286)]^2} = \boxed{1.196}$

accel: $\langle x''(1.286), y''(1.286) \rangle$
$\boxed{= \langle 1.191, -2.879 \rangle}$

1998

$y = x^3 - 3x \qquad x'(t) = (2t+1)^{-1/2} \qquad @\ t=0 \rightarrow x=-4$

a) $x(t) = \int x'(t)\,dt$
$= \int (2t+1)^{-1/2}$
$= \dfrac{(2t+1)^{1/2}}{1/2 \cdot 2} + C$
$= (2t+1)^{1/2} + C$

$x(0) = -4 = (2\cdot 0+1)^{1/2} + C$
$-4 = 1 + C$
$C = -5$
$\boxed{x(t) = (2t+1)^{1/2} - 5}$

b) $y = x^3 - 3x$
$\dfrac{dy}{dt} = 3x^2 \dfrac{dx}{dt} - 3\dfrac{dx}{dt}$
$= (3x^2 - 3)\dfrac{dx}{dt} = \boxed{\left[3\left[(2t+1)^{1/2} - 5\right]^2 - 3\right] \cdot (2t+1)^{-1/2}}$

these are x's!

c) $x(4) = (2\cdot 4+1)^{1/2} - 5$
$= \sqrt{9} - 5$
$= -2$

$y(4) = (-2)^3 - 3(-2)$
$= -8 + 6$
$= -2$

$\boxed{@\ t=4 \rightarrow (-2,-2)}$

$x'(4) = (2\cdot 4+1)^{-1/2} = 1/3$
$y'(4) = \dfrac{3\left[(2\cdot 4+1)^{1/2} - 5\right]^2 - 3}{(2\cdot 4+1)^{1/2}} = \dfrac{3(-2)^2 - 3}{3} = \dfrac{9}{3} = 3$

Speed $= \sqrt{(1/3)^2 + (3)^2}$
$\boxed{= 3.018}$