1999 cont'

c) $x(t) = 0$ ↓ $t = 1.286$

speed: $\sqrt{[x'(1.286)]^2 + [y'(1.286)]^2} = 1.196$

accel: $\langle x''(1.286), y''(1.286) \rangle$

$= \langle 1.191, -2.879 \rangle$

1998

$y = x^3 - 3x$ $\quad x'(t) = (2t+1)^{-1/2}$ $\quad$ @ $t=0 \to x=-4$

a) $x(t) = \int x'(t)\,dt$

$= \int (2t+1)^{-1/2}$

$= \frac{(2t+1)^{1/2}}{1/2 \cdot 2} + C$

$= (2t+1)^{1/2} + C$

$x(0) = -4 = (2\cdot 0 + 1)^{1/2} + C$

$-4 = 1 + C$

$C = -5$

$x(t) = (2t+1)^{1/2} - 5$

b) $y = x^3 - 3x$

$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 3\frac{dx}{dt}$

$= (3x^2 - 3)\frac{dx}{dt} = \left[3\left[(2t+1)^{1/2} - 5\right]^2 - 3\right] \cdot (2t+1)^{-1/2}$

these are x's!

c) $x(4) = (2\cdot 4 + 1)^{1/2} - 5$

$= \sqrt{9} - 5$

$= -2$

$y(4) = (-2)^3 - 3(-2)$

$= -8 + 6$

$= -2$

@ $t=4 \to (-2, -2)$

$x'(4) = (2\cdot 4 + 1)^{-1/2} = 1/3$

$y'(4) = \frac{3\left[(2\cdot 4+1)^{1/2} - 5\right]^2 - 3}{(2\cdot 4+1)^{1/2}} = \frac{3(-2)^2 - 3}{3} = \frac{9}{3} = 3$

Speed $= \sqrt{(1/3)^2 + (3)^2}$

$= 3.018$

1997

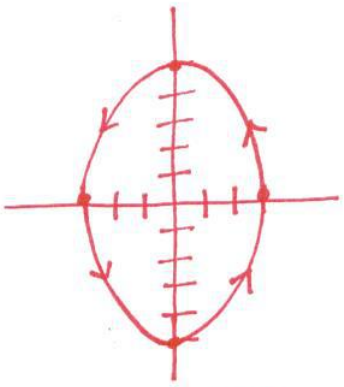
$x(t) = 3\cos(\pi t)$ $y(t) = 5\sin(\pi t)$ $0 \le t \le 6$

a) $x(2.5) = 3\cos(5\pi/2) = 0$

$y(2.5) = 5\sin(5\pi/2) = 5$

@ $t = 2.5 \rightarrow (0, 5)$

b)



c) 3 times

d) $x'(t) = -3\pi\sin(\pi t)$

$y'(t) = 5\pi\cos(\pi t)$

Vel: $\langle -3\pi\sin(\pi t), 5\pi\cos(\pi t) \rangle$

e) distance: $\int_{1.25}^{1.75} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt = 5.392$

1995

$x = t - 2$ $y = (t-2)^2$ (A)

$x = \frac{3}{2}t - 4$ $y = \frac{3}{2}t - 2$ (B)

a) $\text{Vel}_A : \langle 1, 2(t-2) \rangle$

$\text{Vel}_B : \langle 3/2, 3/2 \rangle$

$x'_A = 1$

$y'_A = 2(t-2)$

$\text{Vel}_A : \langle 1, 2 \rangle$ @ $t = 3$

$x'_B = 3/2$

$y'_B = 3/2$

$\text{Vel}_B : \langle 3/2, 3/2 \rangle$ @ $t = 3$

1995 cont'

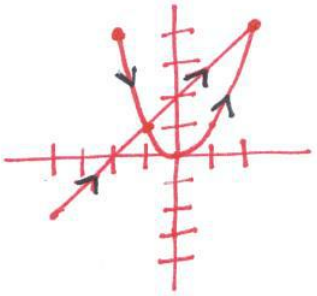
b) $\text{distance} = \int_0^3 \sqrt{1+(2t-4)^2}\,dt$

c) $t-2 = \frac{3}{2}t-4$ $\quad x = 4-2 = 2$

$2 = \frac{1}{2}t$ $\quad y = (4-2)^2 = 4$

$\boxed{t=4}$ $\quad \boxed{@\ t=4 \rightarrow (2,4)}$

d)



1994

$y = \cos x$ $\quad x''(t) = 2$ $\quad @\ t=0 \rightarrow (\pi, -1)$

vel: $\langle 0,0 \rangle$ @ $t=0$

a) $x''(t) = 2$

$x'(t) = 2t + C$

$x'(0) = 2(0) + C = 0$

$C = 0 \rightarrow \boxed{x'(t) = 2t}$ ← y_1

$x(t) = t^2 + C$

$x(0) = (0)^2 + C = \pi$

$C = \pi$

$\boxed{x(t) = t^2 + \pi}$

$\boxed{y(t) = \cos(t^2+\pi)}$

b) $y'(t) = -2t \sin(t^2+\pi)$ ← y_2

$\text{speed} = \sqrt{[x'(\sqrt{4-\pi})]^2 + [y'(\sqrt{4-\pi})]^2}$

@ $t=\sqrt{4-\pi}$ (y_1) (y_2) @ $t=\sqrt{4-\pi}$

$\boxed{\text{Speed} = 2.324}$

Speed when $(4, \cos 4)$

$x = t^2 + \pi = 4$

$t^2 = 4 - \pi$

$t = \sqrt{4-\pi}$

1993

$x(t) = t^2 - 3 \quad y(t) = \frac{2}{3}t^3$

a) $x'(t) = 2t$ ← y_1 $\quad y'(t) = 2t^2$ ← y_2 $\quad |v(5)| = \sqrt{(10)^2 + (50)^2}$

$x'(5) = 10 \quad y'(5) = 50$ $\quad = \sqrt{2600} \approx 50.990$

b) distance $= \int_0^5 \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt = 87.716$

(y_1, y_2)

c) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{2t^2}{2t} = t$

$x = t - 3$

$t^2 = x + 3$

$t = \sqrt{x+3}$

$\therefore \frac{dy}{dx} = \sqrt{x+3}$

1992

$x(t) = e^t \sin t \quad y(t) = e^t \cos t \quad 0 \le t \le 2\pi$

a) $x'(t) = e^t \cos t + e^t \sin t$ ← y_1

$y'(t) = -e^t \sin t + e^t \cos t$ ← y_2

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{e^t \cos t - e^t \sin t}{e^t \cos t + e^t \sin t}$

$= \frac{\cos t - \sin t}{\cos t + \sin t}$

$\frac{dy}{dx}$ @ $t = \pi/2 = \frac{\cos \pi/2 - \sin \pi/2}{\cos \pi/2 + \sin \pi/2} = \frac{-1}{1} = -1$ ($\cos \pi/2 = 0$)

b) speed (y_3): $\sqrt{[x'(1)]^2 + [y'(1)]^2} = 3.844$ @ $t = 1$

(y_1, y_2 @ $t = 1$)

c) distance: $\int_0^1 \underbrace{\sqrt{[x'(t)]^2 + [y'(t)]^2}}_{y_3}\,dt = 2.430$

1991

$v(t) = 12t^2 - 36t + 15$ @ $t=1 \rightarrow (0,0)$

a) $x(t) = \int v(t)$

$x(t) = 4t^3 - 18t^2 + 15t + C$

$x(1) = 0 = 4(1)^3 - 18(1)^2 + 15(1) + C$

$C = -1 \quad \therefore \boxed{x(t) = 4t^3 - 18t^2 + 15t - 1}$

b) $v(t) = 0$

$12t^2 - 36t + 15 = 0$

Quad. Formula

$\boxed{t = 1/2, 5/2}$

c) max vel → accel = 0

$a(t) = 24t - 36$

$t = 3/2$

$v(0) = 15$

$v(3/2) = -12$

$v(2) = -9$

$\therefore$ max velocity is 15 @ $t = 3/2$

d) distance:

$$\int_0^2 |v(t)|\,dt = \int_0^2 |12t^2 - 36t + 15|\,dt = 17$$

1990

$x(t) = (t-1)^3(2t-3)$

a) $v(t) = (t-1)^3(2) + (2t-3)[3(t-1)^2(1)]$

$= (t-1)^2[2(t-1) + 3(2t-3)]$

$= (t-1)^2(8t-11)$

1990 cont

b) $v(t) = (t-1)^2(8t-11)$

Always (+) but $t \neq 1$

$8t-11<0$

$8t<11$

$t<11/8$, $t\neq 1$

but $t \geq 0$ so... $0 \leq t < 1 \cup 1 < t < 11/8$

c) $a(t) = v'(t)$

$= (t-1)^2(8) + (8t-11)[2(t-1)'(1)]$

$= (t-1)[8t-8+16t-22]$

$= (t-1)(24t-30)$

$t=1$ $\quad t = 30/24 = 5/4$

but the particle is at rest $t=1$ so only $t=5/4$ is it moving $\neq a(t)=0$.