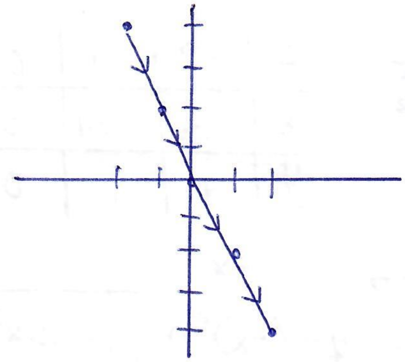
Parametric Packet

1. $x = t$
   $y = -2t$

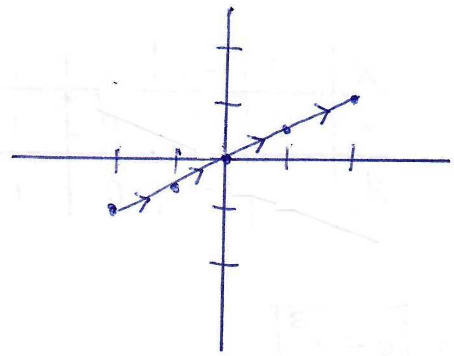
t	-2	-1	0	1	2
x	-2	-1	0	1	2
y	4	2	0	-2	-4



$t = x:\ y = -2t \rightarrow \boxed{y = -2x}$

2. $x = t$
   $y = \frac{1}{2}t$

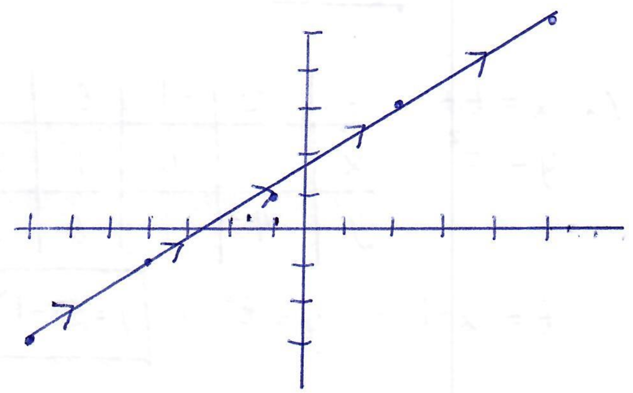
t	-2	-1	0	1	2
x	-2	-1	0	1	2
y	-1	-1/2	0	1/2	1



$t = x:\ y = \frac{1}{2}t \rightarrow \boxed{y = \frac{1}{2}x}$

3. $x = 3t - 1$
   $y = 2t + 1$

t	-2	-1	0	1	2
x	-7	-4	-1	2	5
y	-3	-1	1	3	5

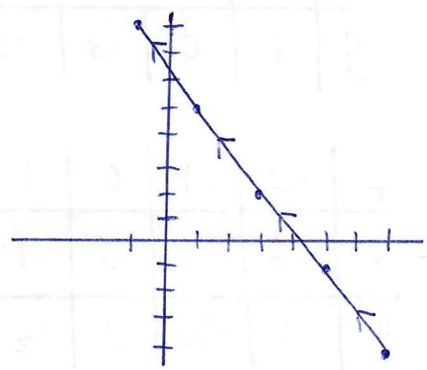


$x = 3t - 1 \rightarrow x + 1 = 3t \rightarrow t = \frac{1}{3}x + \frac{1}{3}$

$y = 2t + 1 \rightarrow y = 2(\frac{1}{3}x + \frac{1}{3}) + 1 \rightarrow y = \frac{2}{3}x + \frac{2}{3} + 1 \rightarrow \boxed{y = \frac{2}{3}x + \frac{5}{3}}$

4. $x = 3 - 2t$
   $y = 2 + 3t$

t	-2	-1	0	1	2
x	7	5	3	1	-1
y	-4	-1	2	5	8



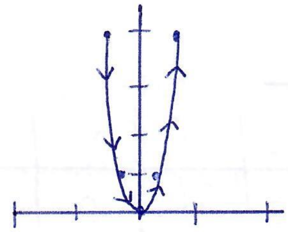
$x = 3 - 2t \rightarrow 2t = 3 - x \rightarrow t = \frac{3}{2} - \frac{1}{2}x$

$y = 2 + 3t \rightarrow y = 2 + 3(\frac{3}{2} - \frac{1}{2}x) \rightarrow y = 2 + \frac{9}{2} - \frac{3}{2}x$

$\boxed{y = -\frac{3}{2}x + \frac{13}{2}}$

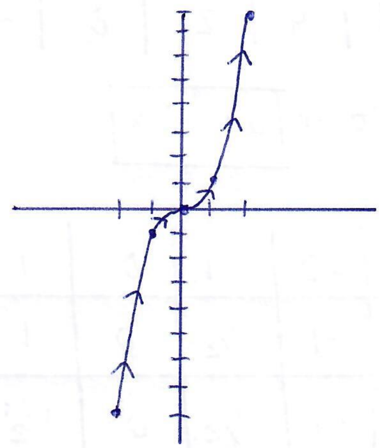
5. $x = \frac{1}{4}t$
$y = t^2$

t	-2	-1	0	1	2
x	$-1/2$	$-1/4$	0	$1/4$	$1/2$
y	4	1	0	1	4



$x = \frac{1}{4}t \rightarrow t = 4x$

$y = t^2 \rightarrow y = (4x)^2 \rightarrow \boxed{y = 16x^2}$

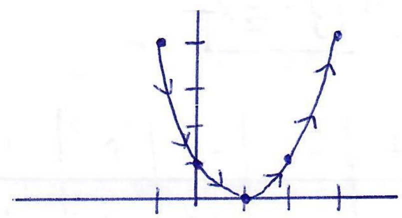


6. $x = t$
$y = t^3$

t	-2	-1	0	1	2
x	-2	-1	0	1	2
y	-8	-1	0	1	8

$x = t$

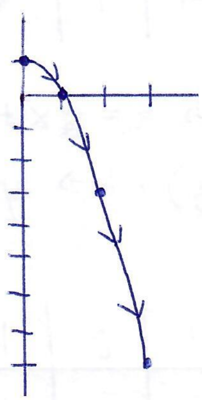
$y = t^3 \rightarrow \boxed{y = x^3}$



7. $x = t+1$
$y = t^2$

t	-2	-1	0	1	2
x	-1	0	1	2	3
y	4	1	0	1	4

$t = x-1 \rightarrow y = t^2 \rightarrow \boxed{y = (x-1)^2}$



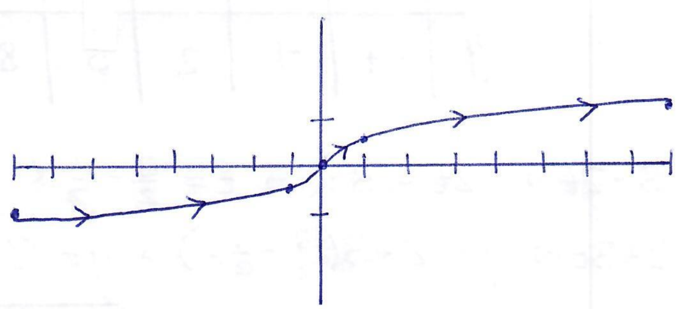
$x = \sqrt{t}$
$t = x^2$
$y = 1-t$
$\boxed{y = 1-x^2}$

8. $x = \sqrt{t}$
$y = 1-t$

t	0	1	4	9
x	0	1	2	3
y	1	0	-3	-8

9. $x = t^3$
$y = \frac{1}{2}t$

t	-2	-1	0	1	2
x	-8	-1	0	1	8
y	-1	$-1/2$	0	$1/2$	1

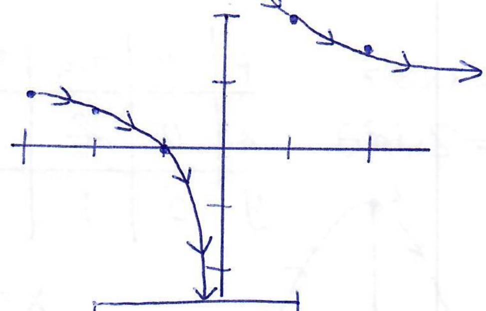


$x = t^3 \rightarrow t = \sqrt[3]{x}$

$y = \frac{1}{2}t \rightarrow \boxed{y = \frac{1}{2}\sqrt[3]{x}}$

10. $x = t - 1$
$y = \frac{t}{t-1}$

t	-2	-1	0	1	2	3
x	-3	-2	-1	0	1	2
y	$2/3$	$1/2$	0	DNE	2	$3/2$

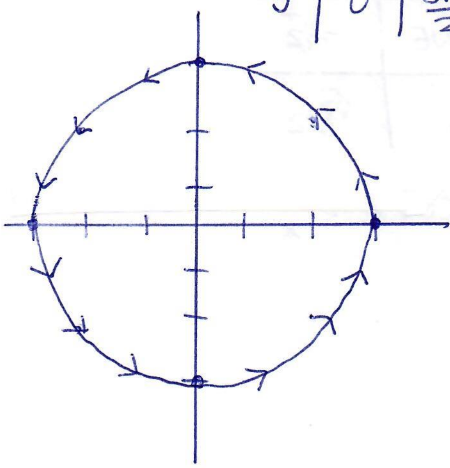


$x = t - 1 \rightarrow t = x + 1$

$y = \frac{t}{t-1} \rightarrow y = \frac{x+1}{x+\cancel{1}-\cancel{1}} \rightarrow y = \frac{x+1}{x} \rightarrow y = \frac{x}{x} + \frac{1}{x} \rightarrow \boxed{y = 1 + \frac{1}{x}}$

11. $x = 3\cos\theta$
$y = 3\sin\theta$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x	3	$\frac{3\sqrt{2}}{2}$	0	$-\frac{3\sqrt{2}}{2}$	-3	$-\frac{3\sqrt{2}}{2}$	0	$\frac{3\sqrt{2}}{2}$
y	0	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0	$-\frac{3\sqrt{2}}{2}$	-3	$-\frac{3\sqrt{2}}{2}$



$x = 3\cos\theta \qquad y = 3\sin\theta$

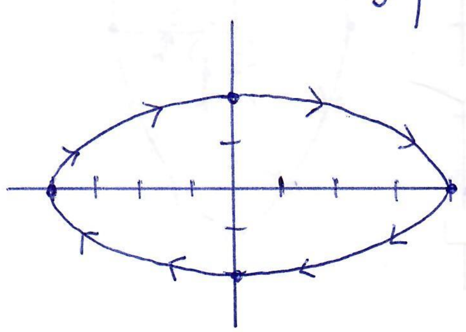
$\cos\theta = \frac{x}{3} \qquad \sin\theta = \frac{y}{3}$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1 \rightarrow \frac{x^2}{9} + \frac{y^2}{9} = 1 \rightarrow \boxed{x^2 + y^2 = 9}$
(Circle)

12. $x = 4\sin 2\theta$
$y = 2\cos 2\theta$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
x	0	4	0	-4	0
y	2	0	-2	0	2



$x = 4\sin 2\theta \qquad y = 2\cos 2\theta$

$\sin 2\theta = \frac{x}{4} \qquad \cos 2\theta = \frac{y}{2}$

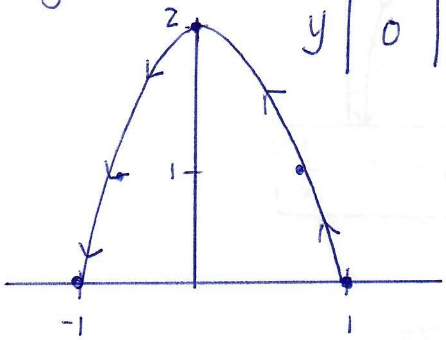
$\sin^2\theta + \cos^2\theta = 1$

$\sin^2 2\theta + \cos^2 2\theta = 1$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \rightarrow \boxed{\frac{x^2}{16} + \frac{y^2}{4} = 1}$
(Ellipse)

13. $x = \cos\theta$
$y = 2\sin^2\theta$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$
y	0	1	2	1	0	1	2	1



$x = \cos\theta \quad y = 2\sin^2\theta$

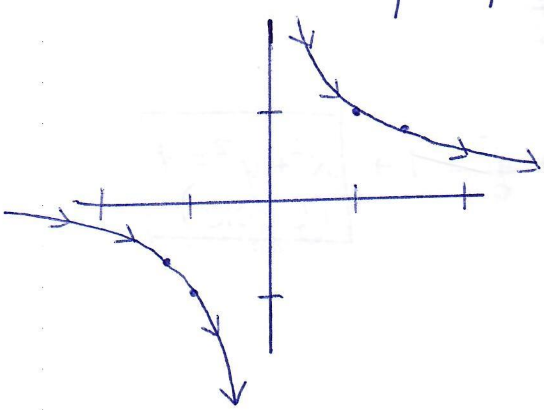
$x^2 = \cos^2\theta \quad \sin^2\theta = \frac{y}{2}$

$\cos^2\theta + \sin^2\theta = 1 \rightarrow x^2 + \frac{y}{2} = 1 \rightarrow \frac{y}{2} = 1 - x^2$

$\boxed{y = 2 - 2x^2}$

14. $x = \sec\theta$
$y = \cos\theta$

t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x	1	$\sqrt{2}$	DNE	$-\sqrt{2}$	-1	$-\sqrt{2}$	DNE	$\sqrt{2}$
y	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$

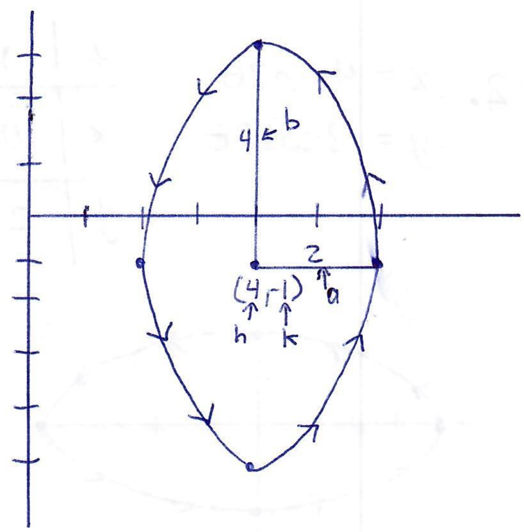


$x = \sec\theta \rightarrow x = \frac{1}{\cos\theta} \rightarrow \cos\theta = \frac{1}{x}$

$y = \cos\theta \rightarrow \boxed{y = \frac{1}{x}}$

15. $x = \overset{h}{4} + \overset{a}{2}\cos\theta$
$y = \underset{k}{-1} + \underset{b}{4}\sin\theta$

θ	0	$\pi/2$	π	$3\pi/2$
x	6	4	2	4
y	-1	3	-1	-5

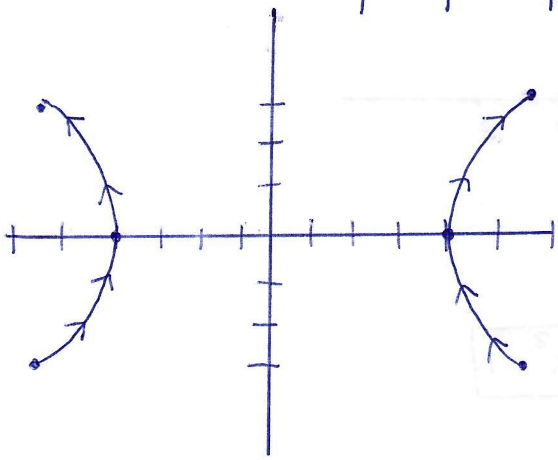


Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\boxed{\frac{(x-4)^2}{4} + \frac{(y+1)^2}{16} = 1}$

16. $x = 4\sec\theta$
$y = 3\tan\theta$

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
x	4	$4\sqrt{2}$	DNE	$-4\sqrt{2}$	-4	$-4\sqrt{2}$	DNE	$4\sqrt{2}$
y	0	3	DNE	-3	0	3	DNE	-3

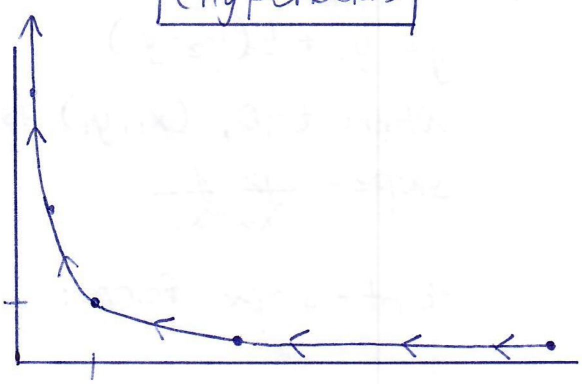


$x = 4\sec\theta \qquad y = 3\tan\theta$
$\sec\theta = \frac{x}{4} \qquad \tan\theta = \frac{y}{3}$

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$
$\sec^2\theta - \tan^2\theta = 1$
$\left(\frac{x}{4}\right)^2 - \left(\frac{y}{3}\right)^2 = 1 \rightarrow \boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1 \text{ (Hyperbola)}}$

17. $x = e^{-t}$
$y = e^{3t}$

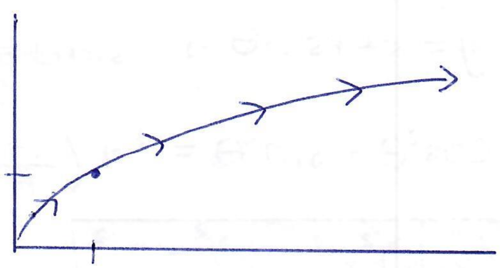
t	-2	-1	0	1	2
x	e^2	e	1	$1/e$	$1/e^2$
y	$1/e^6$	$1/e^3$	1	e^3	e^6



$x = e^{-t}$
$y = e^{3t} = (e^{-t})^{-3} \rightarrow y = x^{-3} \rightarrow \boxed{y = \frac{1}{x^3}}$

18. $x = e^{2t}$
$y = e^t$

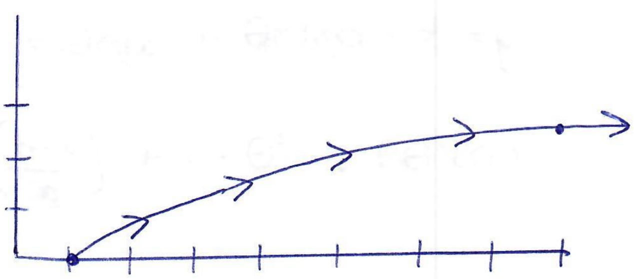
t	-2	-1	0	1	2
x	$1/e^4$	$1/e^2$	1	e^2	e^4
y	$1/e^2$	$1/e$	1	e	e^2



$x = e^{2t}$
$y = e^t = (e^{2t})^{1/2} \rightarrow y = x^{1/2} \rightarrow \boxed{y = \sqrt{x}}$

19. $x = t^3$
$y = 3\ln t$

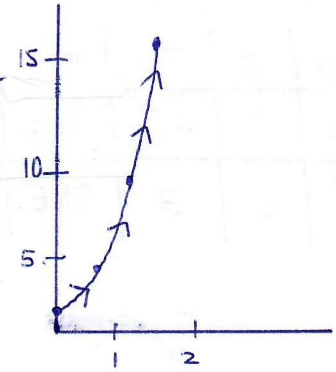
t	1	2	3	4
x	1	8	27	64
y	0	2.1	3.3	4.2



$x = t^3 \rightarrow t = \sqrt[3]{x} = x^{1/3}$
$y = 3\ln t \rightarrow y = 3\ln x^{1/3} \rightarrow y = \cancel{3}\cdot\frac{1}{\cancel{3}}\ln x \rightarrow \boxed{y = \ln x}$

20. $x = \ln t$
$y = t^2$

t	1	2	3	4
x	0	0.7	1.1	1.4
y	1	4	9	16



$e^{x} = \cancel{e}^{\cancel{\ln} t} \rightarrow t = e^x$

$y = t^2 \rightarrow y = (e^x)^2 \rightarrow \boxed{y = e^{2x}}$

21. All a-d represent the line $\boxed{y = 2x+1}$

22. All a-d represent the parabola $\boxed{y = x^2 - 1}$

23. $x = x_1 + t(x_2 - x_1)$
$y = y_1 + t(y_2 - y_1)$
When $t=0$, (x_1, y_1) is the y-intercept.
Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$
Point-slope form: $y - y_1 = m(x - x_1) \rightarrow \boxed{y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$

24. $x = h + r\cos\theta \rightarrow r\cos\theta = x - h \rightarrow \cos\theta = \dfrac{x-h}{r}$
$y = k + r\sin\theta \rightarrow r\sin\theta = y - k \rightarrow \sin\theta = \dfrac{y-k}{r}$
$\cos^2\theta + \sin^2\theta = 1 \rightarrow \left(\dfrac{x-h}{r}\right)^2 + \left(\dfrac{y-k}{r}\right)^2 = 1 \rightarrow \dfrac{(x-h)^2}{r^2} + \dfrac{(y-k)^2}{r^2} = 1$
$\boxed{(x-h)^2 + (y-k)^2 = r^2}$

25. $x = h + a\cos\theta \rightarrow \cos\theta = \dfrac{x-h}{a}$
$y = k + b\sin\theta \rightarrow \sin\theta = \dfrac{y-k}{b}$
$\cos^2\theta + \sin^2\theta = 1 \rightarrow \left(\dfrac{x-h}{a}\right)^2 + \left(\dfrac{y-k}{b}\right)^2 = 1 \rightarrow \boxed{\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1}$

26. $x = h + a\sec\theta \rightarrow \sec\theta = \frac{x-h}{a}$

$y = k + b\tan\theta \rightarrow \tan\theta = \frac{y-k}{b}$

$\sin^2\theta + \cos^2\theta = 1 \rightarrow \tan^2\theta + 1 = \sec^2\theta \rightarrow \sec^2\theta - \tan^2\theta = 1$

$\left(\frac{x-h}{a}\right)^2 - \left(\frac{y-k}{b}\right)^2 = 1 \rightarrow \boxed{\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1}$

27. $\underset{x_1\ y_1}{(0,0)}$ and $\underset{x_2\ y_2}{(5,-2)}$

$x = x_1 + t(x_2 - x_1) \rightarrow x = 0 + t(5-0) \rightarrow \boxed{x = 5t}$

$y = y_1 + t(y_2 - y_1) \rightarrow y = 0 + t(-2-0) \rightarrow \boxed{y = -2t}$

28. $\underset{x_1\ y_1}{(1,4)}$ and $\underset{x_2\ y_2}{(5,-2)}$

$x = x_1 + t(x_2 - x_1) \rightarrow x = 1 + t(5-1) \rightarrow \boxed{x = 1 + 4t}$

$y = y_1 + t(y_2 - y_1) \rightarrow y = 4 + t(-2-4) \rightarrow \boxed{y = 4 - 6t}$

29. Center: $(2,1)$, Radius: 4

$x = h + r\cos\theta \rightarrow \boxed{x = 2 + 4\cos\theta}$

$y = k + r\sin\theta \rightarrow \boxed{y = 1 + 4\sin\theta}$

30. Center: $(-3,1)$, Radius: 3

$x = h + r\cos\theta \rightarrow \boxed{x = -3 + 3\cos\theta}$

$y = k + r\sin\theta \rightarrow \boxed{y = 1 + 3\sin\theta}$

31. Vertices: $(5,0), (-5,0)$; Center: $(0,0)$; Foci: $(4,0), (-4,0)$

$a^2 - b^2 = c^2 \rightarrow 5^2 - b^2 = 4^2 \rightarrow 25 - b^2 = 16 \rightarrow b^2 = 9 \rightarrow b = 3$

$x = h + a\cos\theta \rightarrow x = 0 + 5\cos\theta \rightarrow \boxed{x = 5\cos\theta}$

$y = k + b\sin\theta \rightarrow y = 0 + 3\sin\theta \rightarrow \boxed{y = 3\sin\theta}$

32. Vertices: $(4,7), (4,-3)$; Center: $(4,2)$; Foci: $(4,5), (4,-1)$

$a = 5$, y direction (units from center) $c = 3$ (units from center)

$a^2 - b^2 = c^2 \rightarrow 5^2 - b^2 = 3^2 \rightarrow 25 - b^2 = 9 \rightarrow b^2 = 16 \rightarrow b = 4$, x direction

$x = h + a\cos\theta \rightarrow \boxed{x = 4 + 4\cos\theta}$

$y = k + b\sin\theta \rightarrow \boxed{y = 2 + 5\sin\theta}$

33. Vertices: $(4,0), (-4,0)$; Center: $(0,0)$; Foci: $(5,0), (-5,0)$

$a = 4$, x direction $c = 5$

$a^2 + b^2 = c^2 \rightarrow 4^2 + b^2 = 5^2 \rightarrow 16 + b^2 = 25 \rightarrow b^2 = 9 \rightarrow b = 3$, y direction

$x = h + a\sec\theta \rightarrow x = 0 + 4\sec\theta \rightarrow \boxed{x = 4\sec\theta}$

$y = k + b\tan\theta \rightarrow y = 0 + 3\tan\theta \rightarrow \boxed{y = 3\tan\theta}$

34. Vertices: $(0,1), (0,-1)$; Center: $(0,0)$; Foci: $(0,5), (0,-5)$

$a = 1$, y direction $c = 5$

$a^2 + b^2 = c^2 \rightarrow 1^2 + b^2 = 5^2 \rightarrow 1 + b^2 = 25 \rightarrow b^2 = 24 \rightarrow b = \sqrt{24}$, x direction

$\begin{cases} y = h + a\sec\theta \rightarrow y = 0 + 1\sec\theta \rightarrow \boxed{y = \sec\theta} \\ x = k + b\tan\theta \rightarrow x = 0 + \sqrt{24}\tan\theta \rightarrow \boxed{x = \sqrt{24}\tan\theta} \end{cases}$

Note: switch x & y so the hyperbola will open in the y direction.