Quick Review of Conics

* Circle: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{a^2} = 1$

center: (h,k)
radius: a

* Parabolas: $(x-h)^2 = 4p(y-k)$ opens up/down

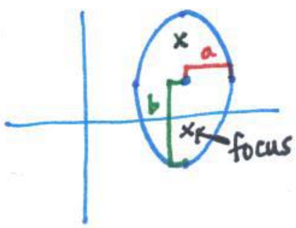
$(y-k)^2 = 4p(x-h)$ opens L/R

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ center: (h,k)
p = distance from center to the focus

* Ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

center: (h,k) foci: $a^2 - b^2 = c^2$

"a" is the distance from center in the x-direction
"b" is the distance from center in the y-direction.



* Hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ opens L/R

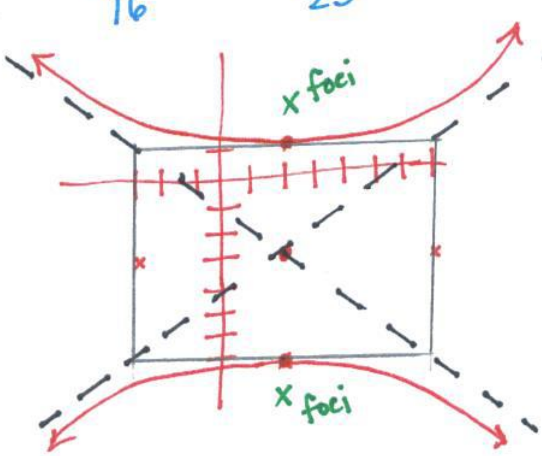
$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2}$ opens up/down

center: (h,k)
foci: $a^2 + b^2 = c^2$

ex: $\dfrac{(y+3)^2}{16} - \dfrac{(x-2)^2}{25} = 1$

center: $(2,-3)$ opens: up/down → vertices
$(2,1)$
$(2,-7)$

y-direction: 4
x-direction: 5



foci: $a^2 + b^2 = c^2$
$16 + 25 = c^2$
$c = \sqrt{41}$

foci: $(2, -3+\sqrt{41})$
$(2, -3-\sqrt{41})$

Parametric Packet

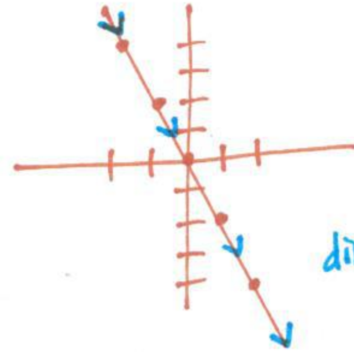
1

$x = t$

$y = -2t$

$\underbrace{\qquad\qquad}_{y=-2x}$

t	x	y
-2	-2	4
-1	-1	2
0	0	0
1	1	-2
2	2	-4



direction: as t incr. y decr.

* Remember: $\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

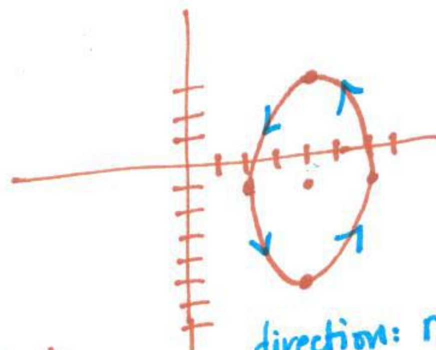
15. $x = 4 + 2\cos\theta$ $\qquad$ $y = -1 + 4\sin\theta$

$\frac{x-4}{2} = \cos\theta$ $\qquad$ $\frac{y+1}{4} = \sin\theta$

$$\left(\frac{x-4}{2}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1$$

$$\frac{(x-4)}{4} + \frac{(y+1)^2}{16} = 1$$

center: $(4,-1)$



direction: rotates like 0 to 2π.

#21

a) $x = t$

$y = 2t + 1$

$\underbrace{\qquad\qquad}_{y=2x+1}$

Line, moves up

b) $x = \cos\theta$

$y = 2\cos\theta + 1$

$\underbrace{\qquad\qquad}_{y=2x+1}$

Line, moves up & down

c) $x = e^{-t}$

$y = 2e^{-t} + 1$

$\underbrace{\qquad\qquad}_{y=2x+1}$

Line, shorter & moves down

d) $x = e^{t}$

$y = 2e^{t} + 1$

$\underbrace{\qquad\qquad}_{y=2x+1}$

Line, longer & moves up

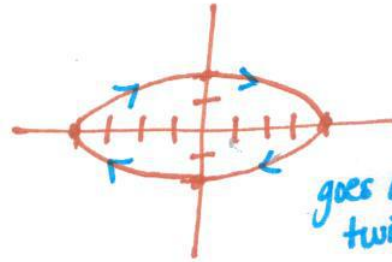
#12 $x = 4\sin 2\theta$ $\quad$ $y = 2\cos 2\theta$

$x = 4\sin u$ $\quad$ $y = 2\cos u$

$\frac{x}{4} = \sin u$ $\quad$ $\frac{y}{2} = \cos u$

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

$$\frac{x^2}{16} + \frac{y^2}{4} = 1 \quad \text{center: } (0,0)$$



goes around twice $\rightarrow 2\theta$

OR

$x = 4\sin 2\theta$ $\quad$ $y = 2\cos 2\theta$

$\frac{x}{4} = \sin 2\theta$ $\quad$ $\frac{y}{2} = \cos 2\theta$

$\downarrow$

$\cos^2\theta = \frac{1+\cos 2\theta}{2} = \frac{1 + y/2}{2} = \frac{2+y}{4}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2} = \frac{1 - y/2}{2} = \frac{2-y}{4}$

$\frac{x}{4} = 2\sin\theta\cos\theta$

$\frac{x^2}{16} = 4\sin^2\theta\cos^2\theta$

$\frac{x^2}{16} = \cancel{4}\left(\frac{2-y}{\cancel{4}}\right)\left(\frac{2+y}{4}\right)$

$\frac{x^2}{16} = \frac{(2-y)(2+y)}{4} = \frac{4-y^2}{4}$

$\frac{x^2}{16} + \frac{y^2}{4} = \frac{4}{4} = 1$ ✓