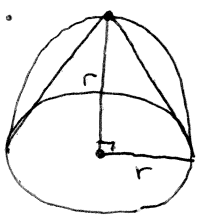
Related Rates AP Questions: 1-8

1.



$\frac{dA}{dt} = 18 \text{ in}^2/\text{s}$ for hemisphere + base

$\frac{dV}{dt} = ?$ for cone when $r = 4$ in

$A = \underbrace{\frac{1}{2} \cdot 4\pi r^2}_{\text{half sphere}} + \underbrace{\pi r^2}_{\text{base}} = 2\pi r^2 + \pi r^2 \rightarrow A = 3\pi r^2$

$\frac{dA}{dt} = 6\pi r \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{dA/dt}{6\pi r} = \frac{18}{6\pi(4)} = \frac{18}{24\pi} = \frac{3}{4\pi}$ in/s

$V = \frac{1}{3}\pi r^2 \cdot r \rightarrow V = \frac{1}{3}\pi r^3$

$\frac{dV}{dt} = \pi r^2 \frac{dr}{dt} = \cancel{\pi} \cdot 4^{\cancel{2}} \cdot \frac{3}{\cancel{4\pi}} = \boxed{12 \text{ in}^3/\text{s}}$

2. $y = 2e^{\cos x}$

a) $\frac{dy}{dx} = 2e^{\cos x}(-\sin x) = \boxed{-2\sin x \cdot e^{\cos x}}$

$\frac{d^2y}{dx^2} = \cancel{-2\sin x \cdot e^{\cos x} \cdot (-\sin x)} + e^{\cos x}(-2\cos x) = 2\sin^2 x\, e^{\cos x} - 2\cos x\, e^{\cos x}$

$\frac{d^2y}{dx^2} = \boxed{2e^{\cos x}(\sin^2 x - \cos x)}$

b) $\frac{dy}{dt} = 5$ units/s, $x = \frac{\pi}{2}$, $\frac{dx}{dt} = ?$

$\frac{dy}{dt} = -2\sin x \cdot e^{\cos x} \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{dy/dt}{-2\sin x \cdot e^{\cos x}} = \frac{5}{-2 \cdot \sin\frac{\pi}{2} \cdot e^{\cos \pi/2}} = \frac{5}{-2 \cdot 1 \cdot \cancel{e^0}} = \boxed{-\frac{5}{2} \text{ units/s}}$

3. a) $3x^2 - y^2 = 23$, $\frac{dy}{dt} = 4$ units/s, $\frac{dx}{dt} = ?$ when $x = 4 \rightarrow 3 \cdot 4^2 - y^2 = 23$

$48 - y^2 = 23$

$y^2 = 25 \rightarrow y = 5$ units

$6x\frac{dx}{dt} - 2y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{2y\, dy/dt}{6x} = \frac{y\, \cancel{dy/dt}}{3x} = \frac{5(\cancel{4})}{3(\cancel{4})} = \boxed{\frac{5}{3} \text{ units/s}}$

3. b) $2x + 9y = k \rightarrow 9y = -2x + k \rightarrow y = \frac{-2}{9}x + \frac{k}{9} \rightarrow m = -2/9$

$m = -2/9$ is $\perp$ to hyperbola when slope of hyperbola $= 9/2$.

$3x^2 - y^2 = 23$

$6x - 2y\frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y}$

$\frac{3x}{y} = \frac{9}{2} \rightarrow 6x = 9y \rightarrow y = \frac{6}{9}x \rightarrow y = \frac{2}{3}x$

$3x^2 - (\frac{2}{3}x)^2 = 23 \rightarrow 3x^2 - \frac{4}{9}x^2 = 23 \rightarrow \frac{23}{9}x^2 = 23 \rightarrow \frac{1}{9}x^2 = 1 \rightarrow x^2 = 9 \rightarrow x = \pm 3$

$y = \frac{2}{3}x$ $\quad y = \frac{2}{3}\cdot 3 = 2 \rightarrow (3,2)$

$\quad y = \frac{2}{3}\cdot -3 = -2 \rightarrow (-3,-2)$

$k = 2x + 9y \rightarrow k = 2(3) + 9(2) \rightarrow k = 6 + 18 \rightarrow \boxed{k = 24}$

$k = 2(-3) + 9(-2) \rightarrow k = -6 - 18 \rightarrow \boxed{k = -24}$

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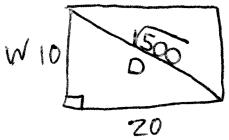
4. $A = 200\ m^2$, $\frac{dL}{dt} = 4\ m/s$, $\frac{dW}{dt} = -0.5\ m/s$, $W = ?$

a) $A = L \cdot W \rightarrow 200 = L \cdot W$

$0 = L\frac{dW}{dt} + W \cdot \frac{dL}{dt} \rightarrow 0 = \frac{200}{W} \cdot \frac{dW}{dt} + W \cdot \frac{dL}{dt} \rightarrow 0 = \frac{200}{W}(-0.5) + W(4)$

$0 = \frac{-100}{W} + 4W \rightarrow \frac{100}{W} = 4W \rightarrow 100 = 4W^2 \rightarrow W^2 = 25 \rightarrow \boxed{W = 5\ m}$

b) $W = 10\ m$, $L = 20\ m$, $D = \sqrt{500}\ m$, $\frac{dL}{dt} = 4\ m/s$



$W^2 + L^2 = D^2$

$2W\frac{dW}{dt} + 2L\frac{dL}{dt} = 2D\frac{dD}{dt}$

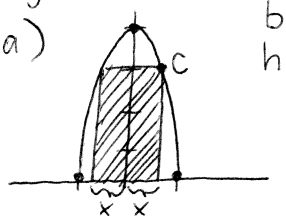
$\frac{dD}{dt} = \frac{W\,dW/dt + L\,dL/dt}{D}$ (need dW/dt)

$0 = L\frac{dW}{dt} + W\frac{dL}{dt} \rightarrow \frac{dW}{dt} = \frac{-W\,dL/dt}{L} = \frac{-10(4)}{20} = \frac{-40}{20} = -2\ m/s$

$\frac{dD}{dt} = \frac{10(-2) + 20(4)}{\sqrt{500}} = \frac{60}{\sqrt{500}} = \boxed{2.683\ m/s}$

5. $y = -4x^2 + 4$

a)



$b = 2x$

$h = 4 - 4x^2$

$A = bh = 2x(4-4x^2) = 8x - 8x^3$

$A' = 8 - 24x^2 = 0 \rightarrow 24x^2 = 8 \rightarrow x^2 = 1/3 \rightarrow x = \sqrt{\frac{1}{3}}$

$y = 4 - 4\left(\frac{1}{3}\right) = 4 - 4\left(\frac{1}{3}\right) = \frac{12}{3} - \frac{4}{3} = \frac{8}{3}$

$C = \boxed{\left(\pm\sqrt{\frac{1}{3}}, \frac{8}{3}\right)}$

b) $\frac{dx}{dt} = 2$ units/s, $\frac{dA}{dt} = ?$ when $x = \frac{1}{2}$

$A = 8x - 8x^3$

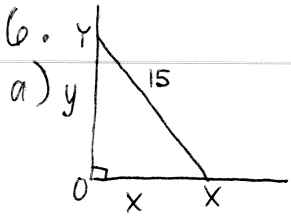
$\frac{dA}{dt} = 8\frac{dx}{dt} - 24x^2\frac{dx}{dt} = 8(2) - 24\left(\frac{1}{2}\right)^2(2) = 16 - 12 = \boxed{4 \text{ units}^2/\text{s}}$

6. $\frac{dx}{dt} = \frac{1}{2}$ ft/s, $x = 9$ ft, $y = 12$ ft, $\frac{dy}{dt} = ?$

3-4-5

9-12-15

a)



$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \rightarrow \frac{dy}{dt} = \frac{-x\,dx/dt}{y} = \frac{-9(1/2)}{12} = \frac{-9}{24} = \boxed{\frac{-3}{8} \text{ ft/s}}$

b) $\frac{dA}{dt} = ?$

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + y\cdot\frac{1}{2}\frac{dx}{dt} = \frac{1}{2}\cdot 9\cdot\frac{-3}{8} + 12\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{-27}{16} + \frac{12}{4} = \frac{-27}{16} + \frac{48}{16} = \boxed{\frac{21}{16} \text{ ft}^2/\text{s}}$

7. $\frac{dV}{dt} = 28\pi \text{ in}^3/\text{s}$, $r = 3$ in, $V = 12\pi \text{ in}^3$, $\frac{dr}{dt} = \frac{1}{2}$ in/s

a) $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi\cdot 3\cdot\frac{1}{2} = \boxed{3\pi \text{ in}^2/\text{s}}$

b) $V = \frac{1}{3}\pi r^2 h \rightarrow h = \frac{3V}{\pi r^2} = \frac{3\cdot 12\pi}{\pi\cdot 3^2} = 4$ in when $r = 3$ in

$V = \frac{1}{3}\pi r^2\cdot h$

$\frac{dV}{dt} = \frac{1}{3}\pi r^2\frac{dh}{dt} + h\cdot\frac{2}{3}\pi r\frac{dr}{dt}$

$\frac{dh}{dt} = \frac{dV/dt - 2/3\,\pi r h\, dr/dt}{\frac{1}{3}\pi r^2} = \frac{28\pi - 2/3\,\pi\cdot 4\cdot 3\cdot 1/2}{\frac{1}{3}\pi\cdot 3^2} = \frac{28-4}{3} = \frac{24}{3} = \boxed{8 \text{ in/s}}$

7. c) $A = \pi r^2$

$$\frac{dA}{dh} = 2\pi r \frac{dr}{dh}$$

$$\frac{dr}{dh} = \frac{dr/dt}{dh/dt} = \frac{1/2}{8} = \frac{1}{16}$$

$$\frac{dA}{dh} = 2\pi \cdot 3 \cdot \frac{1}{16} = \frac{6\pi}{16} = \frac{3\pi}{8} \frac{in^2/s}{in/s} = \boxed{\frac{3\pi}{8} \text{ in}}$$

8. $\frac{dV}{dt} = 261\pi \text{ cm}^3/\text{min}$, $r = 3$ cm, $V = 144\pi \text{ cm}^3$, $\frac{dr}{dt} = 2$ cm/min

a) $V = \pi r^2 h + \frac{4}{3}\pi r^3$

$144\pi = \pi \cdot 3^2 \cdot h + \frac{4}{3}\pi \cdot 3^3 \rightarrow 144\pi = 9\pi h + 36\pi \rightarrow 108\pi = 9\pi h \rightarrow \boxed{h = 12 \text{ cm}}$

b) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt} + 4\pi r^2 \frac{dr}{dt}$

$261\pi = \pi \cdot 3^2 \cdot \frac{dh}{dt} + 12 \cdot 2\pi \cdot 3 \cdot 2 + 4\pi \cdot 3^2 \cdot 2$

$261\pi = 9\pi \frac{dh}{dt} + 144\pi + 72\pi \rightarrow 261\pi = 9\pi \frac{dh}{dt} + 216\pi$

$9\pi \frac{dh}{dt} = 45\pi \rightarrow \frac{dh}{dt} = \frac{45}{9} = \boxed{5 \text{ cm/min}}$