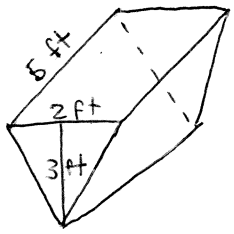
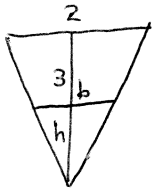
Related Rates AP Questions: 9–15

9. 

$\frac{dV}{dt} = -2\ ft^3/min$

a) $V = \frac{1}{2}bh\ell = \frac{1}{2}\cdot 2\cdot 3\cdot 5 = 15\ ft^3$

b) $\frac{dh}{dt} = ?$ when $V = \frac{1}{4}(15) = 3.75\ ft^3$



$\frac{2}{3} = \frac{b}{h} \rightarrow 2h = 3b \rightarrow b = \frac{2}{3}h$

$V = \frac{1}{2}bh\ell = \frac{1}{2}\cdot\frac{2}{3}h\cdot h\cdot 5 \rightarrow V = \frac{10}{6}h^2 \rightarrow V = \frac{5}{3}h^2 \rightarrow 3.75 = \frac{5}{3}h^2 \rightarrow h = 1.5\ ft$

$\frac{dV}{dt} = \frac{10}{3}h\frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{dV/dt}{10/3\,h} = \frac{-2}{\frac{10}{3}\cdot 1.5} = \boxed{\frac{-2}{5}\ ft/min}$

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c) $\frac{dS}{dt} = ?$ when $V = 3.75\ ft^3$, $h = 1.5\ ft$

$b = \frac{2}{3}h = \frac{2}{3}\cdot\frac{3}{2} = 1\ ft$

$S = b\cdot\ell \rightarrow S = 5b \rightarrow S = 5\cdot\frac{2}{3}h \rightarrow S = \frac{10}{3}h$

$\frac{dS}{dt} = \frac{10}{3}\frac{dh}{dt} = \frac{10}{3}\cdot\frac{-2}{5} = \frac{-20}{15} = \boxed{\frac{-4}{3}\ ft^2/min}$

10. $\frac{dr}{dt} = 0.04\ cm/s$

a) $r = 10\ cm$, $\frac{dV}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} = 4\pi\cdot 10^2\cdot 0.04 = \boxed{16\pi\ cm^3/s}$

b) $V = 36\pi\ cm^3$, $\frac{dA}{dt} = ?$

$36\pi = \frac{4}{3}\pi r^3 \rightarrow r^3 = 27 \rightarrow r = 3\ cm$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi\cdot 3\cdot 0.04 = \boxed{0.24\pi\ cm^2/s}$

c) $\frac{dV}{dt} = \frac{dr}{dt} = 0.04$, $r = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \rightarrow 0.04 = 4\pi r^2\cdot 0.04 \rightarrow r^2 = \frac{1}{4\pi} \rightarrow r = \sqrt{\frac{1}{4\pi}} = \boxed{\frac{1}{2\sqrt{\pi}}\ cm}$

11. $W(v) = 55.6 - 22.1v^{0.16}$

a) $W'(v) = -3.536v^{-0.84}$

$W'(20) = -3.536(20)^{-0.84} =$ **-0.286 °F/mph of wind**

$W'(20)$ is the rate of change of the wind chill temperature when the velocity of the wind is 20 mph.

b) Avg. ROC on $5 \le v \le 60$

$$\frac{W(60) - W(5)}{60 - 5} = \frac{\cancel{55.6} - 22.1(60)^{0.16} - \cancel{55.6} + 22.1(5)^{0.16}}{55} =$$ **-0.254 °F/mph**

$$\frac{-3.536}{v^{0.84}} = -0.254 \rightarrow \left(v^{0.84}\right)^{1/0.84} = (13.932)^{1/0.84} \rightarrow v =$$ **23.011 mph**

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c) $t = 0$, $v = 20$ mph

$\frac{dv}{dt} = 5$ mph/hr, $\frac{dw}{dt} = ?$ at $t = 3$ hr

t	0	1	2	3
v	20	25	30	35

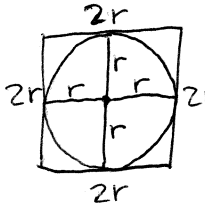
$W = 55.6 - 22.1v^{0.16}$

$$\frac{dW}{dt} = -3.536v^{-0.84}\frac{dv}{dt} = \frac{-3.536}{35^{0.84}} \cdot 5 =$$ **-0.892 °F/hr**

12. $\frac{dC}{dt} = 6$ in/s

a) $\frac{dP}{dt} = ?$

$C = 2\pi r \rightarrow \frac{dC}{dt} = 2\pi\frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{dC/dt}{2\pi} = \frac{6}{2\pi} = \frac{3}{\pi}$ in/s



$P = 8r$

$\frac{dP}{dt} = 8\frac{dr}{dt} = 8 \cdot \frac{3}{\pi} =$ **$\frac{24}{\pi}$ in/s**

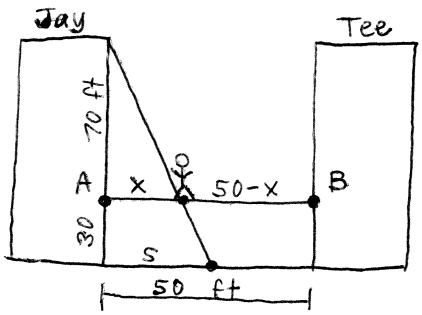
b) $A_0 = 25\pi$ in$^2 \rightarrow r = 5$ in

Area between = square − circle

$A = (2r)^2 - \pi r^2 \rightarrow A = 4r^2 - \pi r^2$

$$\frac{dA}{dt} = 8r\frac{dr}{dt} - 2\pi r\frac{dr}{dt} = 8 \cdot 5 \cdot \frac{3}{\pi} - 2 \cdot \cancel{\pi} \cdot 5 \cdot \frac{3}{\cancel{\pi}} = \frac{120}{\pi} - 30 \approx$$ **8.197 in^2/s**

13.



$\frac{dx}{dt} = 2$ ft/s

a) Midway means $x = 25$ ft, $\frac{ds}{dt} = ?$

$\frac{70}{x} = \frac{100}{s} \rightarrow 70s = 100x \rightarrow s = \frac{100x}{70} = \frac{10}{7}x$

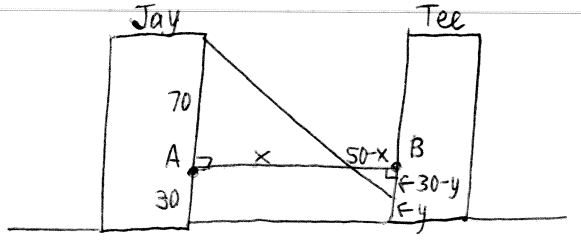
$\frac{ds}{dt} = \frac{10}{7}\frac{dx}{dt} = \frac{10}{7} \cdot 2 = \boxed{\frac{20}{7} \text{ ft/s}}$

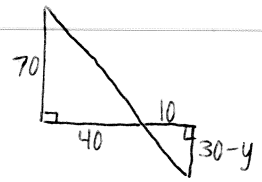
b) $x = ?$ when $s = 50$ ft

$s = \frac{10}{7}x \rightarrow x = \frac{7}{10}s = \frac{7}{10} \cdot 50 = \boxed{35 \text{ ft}}$

c) 10 ft from B $\rightarrow$ $x = 40$ ft

y = shadow up Tee building when $x = 40$:





$\frac{70}{40} = \frac{30-y}{10} \rightarrow 700 = 1200 - 40y$

$40y = 500$

$y = 12.5$ ft when $x = 40$ ft

$\frac{70}{x} = \frac{30-y}{50-x} \rightarrow 3500 - 70x = 30x - xy$

$3500 = 100x - xy$

$0 = 100\frac{dx}{dt} - x\frac{dy}{dx} - y\frac{dx}{dt}$

$x\frac{dy}{dt} = \frac{dx}{dt}(100 - y)$

$\frac{dy}{dt} = \frac{dx/dt\,(100-y)}{x} = \frac{2(100-12.5)}{40} = \boxed{4.375 \text{ ft/s}}$

14. $y = \frac{1}{x^2} = x^{-2}$

a) $(3, 1/9)$

$y' = -2x^{-3} = \frac{-2}{x^3}$ at $x = 3 \rightarrow \frac{-2}{3^3} = \frac{-2}{27}$

$y - \frac{1}{9} = -\frac{2}{27}(x-3) \rightarrow y - \frac{1}{9} = -\frac{2}{27}x + \frac{6}{27} \rightarrow y = -\frac{2}{27}x + \frac{1}{3}$

x-Int when $y = 0 \rightarrow 0 = -\frac{2}{27}x + \frac{1}{3} \rightarrow \frac{2}{27}x = \frac{1}{3} \rightarrow x = \frac{1}{3} \cdot \frac{27}{2} = \boxed{\frac{9}{2} = k}$

14. b) $(w, \frac{1}{w^2})$, $y' = \frac{-2}{w^3}$

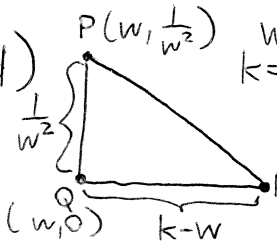
$y - \frac{1}{w^2} = \frac{-2}{w^3}(x - w)$

$y - \frac{1}{w^2} = \frac{-2}{w^3}x + \frac{2}{w^2} \rightarrow y = \frac{-2}{w^3}x + \frac{3}{w^2}$ → x-int when $y = 0$

$0 = \frac{-2}{w^3}x + \frac{3}{w^2} \rightarrow \frac{2}{w^3}x = \frac{3}{w^2} \rightarrow 2x = 3w \rightarrow x = \boxed{\frac{3w}{2} = k}$

c) $\frac{dw}{dt} = 7$ units/s, $w = 5$ units, $\frac{dk}{dt} = ?$

$k = \frac{3}{2}w \rightarrow \frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt} = \frac{3}{2} \cdot 7 = \boxed{\frac{21}{2} \text{ units/s}}$

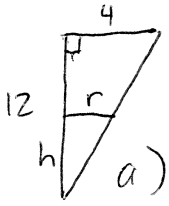
d) 

$w = 5$, $k = \frac{3}{2} \cdot 5 = \frac{15}{2}$ $A = \frac{1}{2}bh = \frac{1}{2}(k-w) \cdot \frac{1}{w^2} = \frac{1}{2w^2}(k-w) = \frac{1}{2}w^{-2}(k-w)$

$\frac{dA}{dt} = \frac{1}{2w^2}\left(\frac{dk}{dt} - \frac{dw}{dt}\right) + (k-w) \cdot \frac{-1}{w^3}\frac{dw}{dt}$

$\frac{dA}{dt} = \frac{1}{2 \cdot 5^2}\left(\frac{21}{2} - 7\right) + (15/2 - 5) \cdot \frac{-1}{5^3} \cdot 7 = \boxed{-0.07 \text{ units}^2/\text{s}}$ (decreasing)

15. $h = 12$ ft, $d = 8$ ft, $r = 4$ ft, $0 = \underset{r = 20 \text{ ft}}{\text{base}}$ 400π ft^2, $\frac{dh}{dt} = (h - 12)$ ft/min



$\frac{4}{12} = \frac{r}{h} \rightarrow 4h = 12r$

$r = \frac{1}{3}h$

a) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{3}\pi \cdot \frac{1}{9}h^2 \cdot h \rightarrow \boxed{V = \frac{1}{27}\pi h^3}$

b) $\frac{dV}{dt} = ?$ when $h = 3$ ft

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt} = \frac{1}{9} \cdot \pi \cdot 3^2 \cdot (h - 12) = \pi(3 - 12) = \boxed{-9\pi \text{ ft}^3/\text{min}}$

c) $V = \pi r^2 h = \pi \cdot 20^2 \cdot h \rightarrow V = 400\pi h = 400\pi y$

$\frac{dV}{dt} = 400\pi \frac{dy}{dt} \rightarrow \frac{dy}{dt} = \frac{dV/dt}{400\pi} = \frac{9\pi}{400\pi} = \frac{9}{400} = \boxed{0.0225 \text{ ft/min}}$