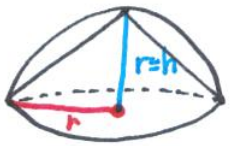
Related Rates Packet

1.



$r = 4$ in

$\frac{dV_{cone}}{dt} = ?$

$\frac{dSA}{dt} = 18 \text{ in}^2/\text{sec}$

$V = \frac{\pi}{3} r^2 h \rightarrow$ cone

$V = \frac{\pi}{3} r^3$

$\frac{dV}{dt} = \frac{\pi}{3} \cdot 3r^2 \frac{dr}{dt}$

$\frac{dV_{cone}}{dt} = \pi r^2 \frac{dr}{dt}$

$\frac{dV_{cone}}{dt} = \pi (4)^2 \left(\frac{3}{4\pi}\right)$

$\boxed{\frac{dV_{cone}}{dt} = 12 \text{ in}^3/\text{sec}}$

$SA = 2\pi r^2 + \pi r^2 = 3\pi r^2$

hemisphere, circle base

$\frac{dSA}{dt} = 6\pi r \frac{dr}{dt}$

$18 = 6\pi (4) \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{18}{24\pi} = \frac{3}{4\pi}$

2. $y = 2e^{\cos x}$

a) $\frac{dy}{dx} = 2e^{\cos x} \cdot (-\sin x) = \boxed{-2\sin x\, e^{\cos x}}$

$\frac{d^2y}{dx^2} = +2\sin x \left(+\sin x\, e^{\cos x}\right) + e^{\cos x} \left(-2\cos x\right)$

$\boxed{= 2e^{\cos x}\left(\sin^2 x - \cos x\right)}$

b) $\frac{dy}{dt} = 5$ units/sec

$x = \pi/2$

$\frac{dx}{dt} = ?$

$\frac{dy}{dt} = -2\sin x\, e^{\cos x} \cdot \frac{dx}{dt}$

$5 = \left(-2 \sin \pi/2 \; e^{\cos \pi/2}\right) dx/dt$

$5 = -2 \frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = -\frac{5}{2} \text{ units/sec}}$

3a) $3x^2 - y^2 = 23$

$6x \frac{dx}{dt} - 2y \frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{2y \, dy/dt}{6x}$

$\frac{dx}{dt} = \frac{(5)(4)}{3(4)} = \boxed{\frac{5}{3} \text{ units/sec}}$

$\frac{dy}{dt} = 4$ units/sec

$x = 4$

$\frac{dx}{dt} = ?$

$3(4)^2 - y^2 = 23$

$-y^2 = -25$

$y = 5$

36) $2x + 9y = -k$ $\quad$ $3x^2 - y^2 = 23$

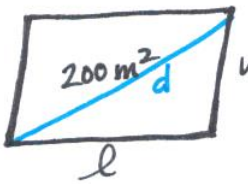
$m = -\frac{2}{9}$ $\quad \perp m = \frac{9}{2}$ $\quad$ $6x\frac{dx}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y}$

$\frac{3x}{y} = \frac{9}{2}$

$x = \pm 3$, $y = \pm 2$ $\rightarrow$ $2(\pm 3) + 9(\pm 2) = -k$

$\boxed{\pm 24 = k}$

4a) 

$\frac{d\ell}{dt} = 4$ m/sec

$w = ?$

$\frac{dw}{dt} = -0.5$ m/sec

$A = \ell \cdot w \rightarrow 200 = \ell \cdot w$ so $\ell = \frac{200}{w}$

$0 = \ell\frac{dw}{dt} + w \cdot \frac{d\ell}{dt}$

$0 = \frac{200}{w}(-0.5) + w(4)$

$0 = -\frac{100}{w} + 4w$

$0 = \frac{4}{w}(-25 + w^2)$

$w = 0$ $\quad$ $w = \pm 5$ $\quad \therefore \boxed{w = 5}$

b) $w = \frac{200}{\ell}$

$w = 10$, $\ell = 20$

$\frac{dd}{dt} = ?$

$d = \sqrt{500}$ (by Pyth.)

$w^2 + \ell^2 = d^2$

$\left(\frac{200}{\ell}\right)^2 + \ell^2 = d^2$

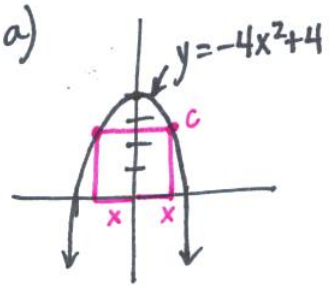
$\frac{40000}{\ell^2} + \ell^2 = d^2$

$\left(-80000\,\ell^{-3} + 2\ell\right)\frac{d\ell}{dt} = 2d\,\frac{dd}{dt}$

$\frac{dd}{dt} = \frac{\left(-40000\,\ell^{-3} + \ell\right)\frac{d\ell}{dt}}{d} = \frac{\left[40{,}000\,(20)^{-3} + 20\right]4}{\sqrt{500}}$

$\boxed{\frac{dd}{dt} = \frac{120}{2\sqrt{500}} \approx 2.683 \text{ m/sec}}$

5. a)



$A = 2x(-4x^2+4)$

$A = -8x^3+8x$

$A' = -24x^2+8 = 0$

$-8(3x^2+1) = 0$

$x = \pm\frac{1}{\sqrt{3}} \approx .577$ $\therefore$ $\boxed{C = (\pm.577, 2.668)}$

b) $\frac{dx}{dt} = 2$ units/sec

$x = .5$

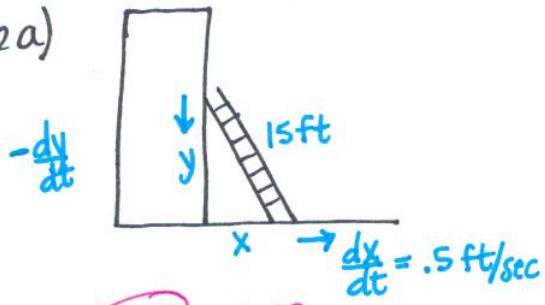
$\frac{dA}{dt} = ?$

$\frac{dA}{dt} = -24x^2\frac{dx}{dt} + 8\frac{dx}{dt}$

$\frac{dA}{dt} = -24(.5)^2(2) + 8(2)$

$\boxed{\frac{dA}{dt} = 4 \text{ units}^2/\text{sec}}$

6 a)



$x = 9 \rightarrow y = 12$

$\frac{dy}{dt} = ?$

$x^2+y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x\,dx/dt}{y} = \frac{-9(.5)}{12}$

$\boxed{\frac{dy}{dt} = -.375 \text{ ft/sec}}$

b) $A = \frac{1}{2}x\cdot y$ $\quad \frac{dA}{dt} = ?$ $\quad x = 9$

$\frac{dA}{dt} = \frac{x}{2}\cdot\frac{dy}{dt} + y\left(\frac{1}{2}\frac{dx}{dt}\right)$

$= \frac{9}{2}(-.375) + \frac{12}{2}(.5)$

$\boxed{\frac{dA}{dt} = 1.3125 \text{ ft}^2/\text{sec}}$

7. $V = \frac{\pi}{3} r^2 h$ $\quad \frac{dV}{dt} = 28\pi \text{ in}^3/\text{sec}$

a) $\frac{dA}{dt} = ?$

$r = 3$ in, $V = 12\pi$ in^3 → $12\pi = \frac{\pi}{3}(3)^2 h$, $h = 4$

$\frac{dr}{dt} = .5$ in/sec

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(3)(.5)$

$\frac{dA}{dt} = 3\pi \text{ in}^2/\text{sec}$

b) $V = \frac{\pi}{3} r^2 h$

$\frac{dV}{dt} = \frac{\pi}{3} r^2 \frac{dh}{dt} + h \cdot \frac{2\pi}{3} r \frac{dr}{dt}$

$28\pi = \frac{\pi}{3}(3)^2 \frac{dh}{dt} + 4 \cdot \frac{2\pi}{3}(3)(.5)$

$\frac{28\pi - 4\pi}{3\pi} = \frac{3\pi \, dh/dt}{3\pi}$

$8 \text{ in/sec} = \frac{dh}{dt}$

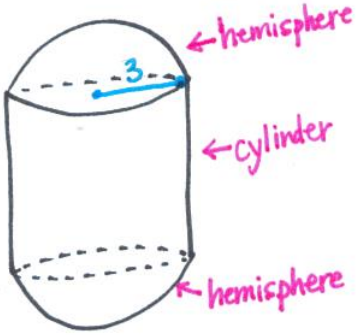
c) $\frac{dA}{dh} = ?$ $\quad A = \pi r^2$

$\frac{dA}{dh} = 2\pi r \frac{dr}{dh}$ → $\frac{dr/dt}{dh/dt} = \frac{.5 \text{ in/sec}}{8 \text{ in/sec}} = .0625$ or $\frac{1}{16}$

$\frac{dA}{dh} = 2\pi(3)(.0625)$

$\frac{dA}{dh} = .375\pi \text{ in}$ or $\frac{3\pi}{8}$ in

8.



$r = 3$ cm

$V = 144\pi$ cm^3

$\frac{dr}{dt} = 2$ cm/min

$\frac{dV}{dt} = 261\pi$ cm^3/min

V = sphere + cylinder

$V = \frac{4\pi}{3} r^3 + \pi r^2 h$

a) $h = ?$

$144\pi = \frac{4\pi}{3}(3)^3 + \pi(3)^2 h$

$144\pi \quad 36\pi + 9\pi h$

$108 = 9h$ $\quad h = 12$ cm

b) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} + \pi r^2 \frac{dh}{dt} + h \cdot 2\pi r \frac{dr}{dt}$

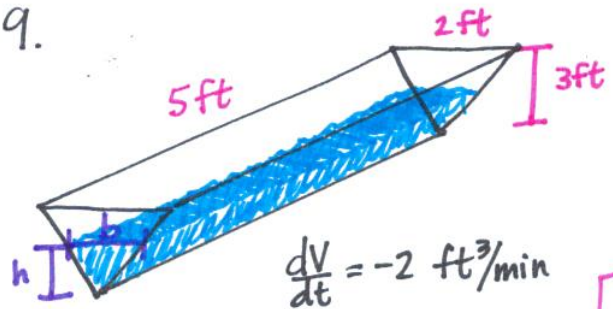
$261\pi = 4\pi(3)^2(2) + \pi(3)^2 \frac{dh}{dt} + (12) \cdot 2\pi(3)(2)$

$261 = 72 + 9 \frac{dh}{dt} + 144$

$45 = 9 \, dh/dt$

$\frac{dh}{dt} = 5$ cm/min

9.



$\frac{dV}{dt} = -2 \text{ ft}^3/\text{min}$

a) $V = B \cdot l$

$V = (\frac{1}{2} b \cdot h)(5)$

$V = \frac{1}{2}(2)(3)(5) = 15 \text{ ft}^3$

$\frac{2}{3} = \frac{b}{h}$

$2h = 3b$

$b = \frac{2}{3}h$

b) $\frac{dh}{dt} = ?$

$V = \frac{15}{4} \text{ ft}^3$

$h = 3/2 \text{ ft}$

$V = \frac{1}{2}(\frac{2}{3}h)(h)(5) = \frac{5}{3}h^2$

$\frac{dV}{dt} = \frac{10}{3} h \frac{dh}{dt}$

$-2 = \frac{10}{3}(\frac{3}{2}) \frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{2}{5} \text{ ft/sec}$

$\frac{15}{4} = \frac{5}{2} b \cdot h$

$\frac{15}{4} = \frac{5}{2}(\frac{2}{3}h)h$

$\frac{9}{4} = h^2$

$h = 3/2$

c) $SA = b \cdot l = b \cdot (5)$

$\frac{dSA}{dt} = 5 \frac{db}{dt}$

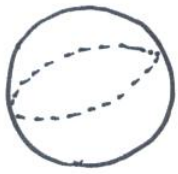
$= 5(\frac{-4}{15})$

$\frac{dSA}{dt} = -\frac{4}{3} \text{ ft}^2/\text{sec}$

$b = \frac{2}{3}h$

$\frac{db}{dt} = \frac{2}{3}\frac{dh}{dt} = \frac{2}{3}(\frac{-2}{5}) = \frac{-4}{15}$

10.



$\frac{dr}{dt} = .04 \text{ cm/sec}$

a) $r = 10 \text{ cm}$ $V = \frac{4\pi}{3} r^3$

$\frac{dV}{dt} = ?$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (10)^2 (.04)$

$\frac{dV}{dt} = 16\pi \text{ cm}^3/\text{sec}$

b) $V = 36\pi \text{ cm}^3 = \frac{4\pi}{3} r^3$

$27 = r^3$

$r = 3$

$\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi (3)(.04)$

$= .24\pi \text{ cm}^2/\text{sec}$

$\approx .754 \text{ cm}^2/\text{sec}$

c) $\frac{dV}{dt} = \frac{dr}{dt}$ $r = ?$

$V = \frac{4\pi}{3} r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ (equal!)

$1 = 4\pi r^2$

$\frac{1}{4\pi} = r^2 \rightarrow$ $r = \frac{1}{2\sqrt{\pi}} \approx .282 \text{ cm}$

11. $w(v) = 55.6 - 22.1 v^{0.16}$ $\quad 5 \le v \le 60$

a) $w'(v) = -22.1(0.16)v^{-.84}$

$w'(20) = -3.536(20)^{-.84}$

$\approx .286\,^\circ F$

b) ave ROC $= \dfrac{w(60)-w(5)}{60-5}$

$= \dfrac{13.050 - 27.009}{55}$ $\quad \approx -.254\,^\circ F$

c) $t=0$; $V = 20$ mph

$\dfrac{dV}{dt} = 5$ mph

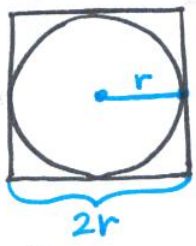
$\dfrac{dw}{dt} = ?$ @ $t = 3$ hrs ; $V = 35$ mph

$\dfrac{dw}{dt} = -3.536 V^{-.84} \dfrac{dV}{dt}$

$= -3.536(35)^{-.84}(5)$

$\dfrac{dw}{dt} \approx -.892\,^\circ F/hr$

12.



$\dfrac{dC}{dt} = 6$ in/sec

$P = 4(2r)$

$P = 8r$

$C = 2\pi r$

a) $\dfrac{dP}{dt} = ?$

$\dfrac{dP}{dt} = 8\dfrac{dr}{dt}$

$\dfrac{dP}{dt} = 8\left(\dfrac{3}{\pi}\right)$

$\dfrac{dP}{dt} = \dfrac{24}{\pi}$ in/sec ≈ 7.639 in/sec

$\dfrac{dC}{dt} = 2\pi \dfrac{dr}{dt}$

$6 = 2\pi \dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{3}{\pi}$

b) $A = 25\pi$ in^2 (circle)

$25\pi = \pi r^2$

$r^2 = 25$

$r = 5$

$A_2 = (2r)^2 - \pi r^2$ (Between circle & square)

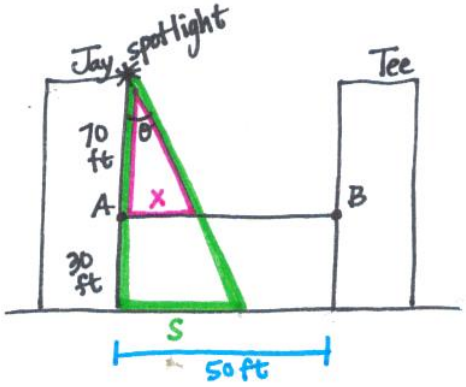
$A_2 = 4r^2 - \pi r^2 = r^2(4-\pi)$

$\dfrac{dA_2}{dt} = (4-\pi)\,2r\dfrac{dr}{dt}$

$= (4-\pi)(2)(5)(3/\pi)$

$\dfrac{dA_2}{dt} \approx 8.197$ in^2/sec

13.



$\frac{dx}{dt} = 2$ ft/sec

a) $x = 25$ ft

$\frac{ds}{dt} = ?$

$\frac{70}{100} = \frac{x}{s}$

$70s = 100x$

$70\frac{ds}{dt} = 100\frac{dx}{dt}$

$\frac{ds}{dt} = \frac{100(2)}{70}$

$\boxed{\frac{ds}{dt} = 2.857 \frac{ft}{sec}}$

b) $\frac{70}{100} = \frac{x}{50}$

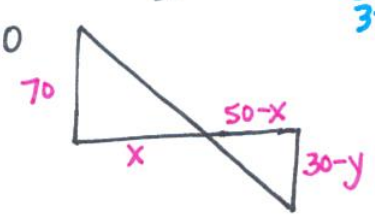
$3500 = 100x$

$\boxed{x = 35 \text{ ft}}$

c) $x = 40$ $\quad \frac{dy}{dt} = ?$

$x = 35 \rightarrow y = 0 \rightarrow s = 111.803$

$x = 50 \rightarrow y = 30$



$\frac{70}{30-y} = \frac{x}{50-x}$

$\frac{70}{30-y} = \frac{40}{10}$

$70 = 120 - 4y$

$4y = 50$

$y = 12.5$

$3500 - 70x = 30x - xy \rightarrow 3500 = 100x - xy$

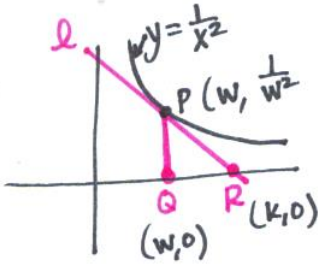
$0 = 100\frac{dx}{dt} - \left[x\frac{dy}{dt} + y \cdot \frac{dx}{dt}\right]$

$0 = 100(2) - \left[40\frac{dy}{dt} + 12.5(2)\right]$

$\frac{40\frac{dy}{dt}}{40} = \frac{200 - 25}{40}$

$\boxed{\frac{dy}{dt} = 4.375 \text{ ft/sec}}$

14.



a) $w = 3$

$P = (3, 1/9)$

$Q = (3, 0)$

$R = (k, 0)$

$m = \frac{0 - 1/9}{k - 3} = \frac{-1}{9(k-3)} = \frac{-2}{27}$

$y' = -2x^{-3}$ @ $x = 3$

$y' = \frac{-2}{27}$

$-27 = -18(k-3)$

$1.5 = k - 3$

$\boxed{4.5 = k}$

14b) $(w, 1/w^2)$ $(k, 0)$

$m = \frac{0 - w^{-2}}{k - w}$ $\quad y = w^{-2}$ $\quad y' = -2w^{-3}$

$m = \frac{-1}{w^2(k-w)} = \frac{-2}{w^3}$

$-w^3 = -2w^2k + 2w^3$

$2w^2k = 3w^3$

$\boxed{k = \frac{3w}{2}}$

c) $\frac{dw}{dt} = 7$ units/sec

$w = 5$ units

$\frac{dk}{dt} = ?$

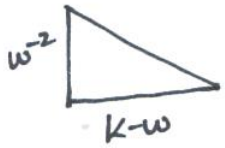
$k = \frac{3}{2}w$

$\frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt}$

$= \frac{3}{2}(7)$

$\boxed{\frac{dk}{dt} = \frac{21}{2} \text{ units/sec}}$

d) $\frac{dA}{dt} = ?$



$A = \frac{1}{2}(k-w)(w^{-2})$

$= \frac{1}{2}(1.5w - w)(w^{-2})$

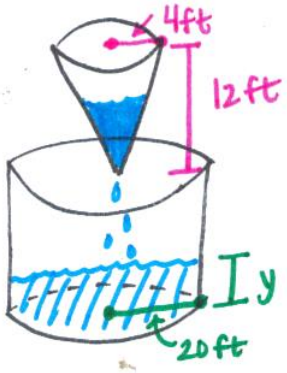
$A = \frac{-1}{4}w^{-1}$

$\frac{dA}{dt} = \frac{1}{4}w^{-2}\frac{dw}{dt}$

$= \frac{1}{4}(5)^{-2}(7)$

$\boxed{\frac{dA}{dt} = \frac{7}{100} \text{ units}^2/\text{sec} \therefore \text{Area is decreasing}}$

15.



Cylinder Base Area $= 400\pi\ ft^2$

$400\pi = \pi r^2$

$400 = r^2$

$r = 20$

$\frac{dh}{dt} = (h-12)\ ft/min$

a) $\frac{4}{12} = \frac{r}{h}$

$4h = 12r$

$r = \frac{h}{3}$

$V = \frac{\pi}{3}r^2h \rightarrow V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 \cdot h$

$$\boxed{V = \frac{\pi}{27}h^3}$$

b) $\frac{dV}{dt} = ?$

$h = 3$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{9}(3)^2(3-12)$

$$\boxed{\frac{dV}{dt} = -9\pi\ ft^3/min}$$

c) Volume of Cylinder: $V = \pi r^2 y$

$V = \pi(20)^2 y = 400\pi y$

$\frac{dV}{dt} = 400\pi\frac{dy}{dt}$

$\frac{-9\pi}{400\pi} = \frac{400\pi\ dy/dt}{400\pi}$

$$\boxed{\frac{dy}{dt} = -\frac{9}{400} \approx .0225\ ft/min}$$