Taylor Series (Section 10.2)

* A Taylor polynomial: approximates a function using a polynomial.

$$T(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots + a_nx^n$$

$a_0 = T(0)$, $a_1 = \frac{T'(0)}{1!}$, $a_2 = \frac{T''(0)}{2!}$, $a_3 = \frac{T'''(0)}{3!}$, $a_4 = \frac{T^{4}(0)}{4!}$, $\dots$, $a_n = \frac{T^{n}(0)}{n!}$

← centered @ $x=0$

ex: $\ln(1+x)$ Find a 4th degree polynomial. (Not using memorization!)

ex: Construct a 7th order Taylor Poly for $\sin x$. (Not using memorization!)

* A Taylor series centered @ $x=0$ is called a "Maclaurin Series" pg 495

$$T(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots + \frac{f^n(0)x^n}{n!} = \sum_{k=0}^{\infty} \frac{f^k(0)x^k}{k!}$$

** If it is NOT centered @ $x=0$, but centered @ $x=a$, then plug in $(x-a)$ in for x.

ex: Write a Taylor series centered @ $x=2$ for $f(x) = e^x$.

ex: Find a 3rd order Taylor polynomial for $f(x) = 2x^3 - 3x^2 + 4x - 5$ centered @ $x=1$.