# Section 10.4: QR: 1-10, Ex: 7-21 odd

1. $\lim\limits_{n\to\infty} \frac{n}{n+1} \cdot |x| = 1 \cdot |x| = \boxed{|x|}$

2. $\lim\limits_{n\to\infty} \frac{n^2}{n^2-n} \cdot |x-3| = 1 \cdot |x-3| = \boxed{|x-3|}$

3. $\lim\limits_{n\to\infty} \frac{|x|^n}{n!} = \frac{|x|^\infty}{\infty!} = \frac{x\cdot x\cdot x\cdot x\cdot \cdots}{\infty(\infty-1)(\infty-2)} = \frac{\text{big}}{\text{bigger}} = \boxed{0}$

4. $\lim\limits_{n\to\infty} \frac{(n+1)^4}{(2n)^4} x^2$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$\lim\limits_{n\to\infty} \frac{n^4+4n^3+6n^2+4n+1}{16n^4} x^2 = \boxed{\frac{1}{16}x^2}$

5. $\lim\limits_{n\to\infty} \frac{|2x+1|^{n+1} \cdot 2^n}{2^{n+1}\cdot |2x+1|^n} = |2x+1|^{\cancel{n}+1-\cancel{n}} \cdot 2^{\cancel{n}-\cancel{n}-1} = |2x+1|^1 \cdot 2^{-1} = \boxed{\frac{|2x+1|}{2}}$

6. $\sum 5n$, $\sum n^2$

| | | |
|---|---|---|
| $n=1$ | 5 | 1 |
| $n=2$ | 10 | 4 |
| $n=3$ | 15 | 9 |
| $n=4$ | 20 | 16 |
| $n=5$ | 25 | 25 |
| $n=6$ | 30 | 36 |
| $n=7$ | 35 | 49 |

$\boxed{a_n = n^2,\ b_n = 5n,\ N = 6}$

7. $\sum n^5$, $\sum 5^n$

| | | |
|---|---|---|
| $n=1$ | 1 | 5 |
| $n=2$ | 32 | 25 |
| $n=3$ | 243 | 125 |
| $n=4$ | 1,024 | 625 |
| $n=5$ | 3,125 | 3125 |
| $n=6$ | 7,776 | 15,625 |
| $n=7$ | 16,807 | 78,125 |

$\boxed{a_n = 5^n,\ b_n = n^5,\ N = 6}$

8. $\sum \ln n$, $\sum \sqrt{n}$

Make a table: $\boxed{a_n = \sqrt{n},\ b_n = \ln(n),\ N = 1}$

9. $\sum \frac{1}{10^n}$, $\sum \frac{1}{n!}$

Make a table: $\boxed{a_n = \frac{1}{10^n},\ b_n = \frac{1}{n!},\ N = 25}$

10. $\sum \frac{1}{n^2}$, $\sum \frac{1}{n^3}$

Make a table: $\boxed{a_n = \frac{1}{n^2},\ b_n = \frac{1}{n^3},\ N = 2}$

Exercises:

7. $\sum_{n=0}^{\infty} \underset{\uparrow\, r=x}{x}^{n}$ $\quad |x|<1 \to \boxed{R=1}$

9. $\sum_{n=0}^{\infty} (-1)^n(4x+1)^n = (\underbrace{-1(4x+1)}_{r})^n$ $\quad |-1(4x+1)|<1$

$|4x+1|<1$

$|x+1/4|<1/4 \to \boxed{R=1/4}$

11. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n} = \Big(\underbrace{\frac{x-2}{10}}_{r}\Big)^n$ $\quad \left|\frac{x-2}{10}\right|<1 \to |x-2|<10 \to \boxed{R=10}$

13. $\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\cdot 3^n}$ $\quad$ Ratio Test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$\lim_{n\to\infty} \frac{|x|^{n+1}}{(n+1)\sqrt{n+1}\cdot 3^{n+1}} \cdot \frac{3^n\cdot n\sqrt{n}}{|x|^n} = \lim_{n\to\infty} \frac{n\sqrt{n}}{(n+1)\sqrt{n+1}} \cdot |x|^{\cancel{n}+1-\cancel{n}} \cdot 3^{\cancel{n}-\cancel{n}-1}$$

$$\lim_{n\to\infty} \frac{\cancel{n^{3/2}}}{\cancel{n^{3/2}}} \cdot |x|^1 \cdot 3^{-1} = 1\cdot\frac{|x|}{3} = \frac{|x|}{3} < 1 \to |x|<3 \to \boxed{R=3}$$

15. $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$

$$\lim_{n\to\infty} \frac{(n+1)|x+3|^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n\,|x+3|^n} = \lim_{n\to\infty} \frac{n+1}{n} \cdot |x+3|^{\cancel{n}+1-\cancel{n}} \cdot 5^{\cancel{n}-\cancel{n}-1}$$

$$1\cdot |x+3|^1 \cdot 5^{-1} = \frac{|x+3|}{5} < 1 \to |x+3|<5 \to \boxed{R=5}$$

17. $\sum_{n=0}^{\infty} n!\,(x-4)^n$

$$\lim_{n\to\infty} \frac{(n+1)!\,|x-4|^{n+1}}{n!\,|x-4|^n} = \lim_{n\to\infty} (n+1)\,|x-4|^{\cancel{n}+1-\cancel{n}} = \infty \cdot |x-4| = \infty$$

$\infty$ is never less than 1, so the series diverges unless $x=4$.

No $x$ values converge except $x=4$, so $\boxed{R=0}$

19. $\sum_{n=0}^{\infty} (-2)^n (n+1)(x-1)^n$

$$\lim_{n\to\infty} \frac{(-2)^{n+1}(n+2)\,|x-1|^{n+1}}{(-2)^n (n)\,|x-1|^n} = \lim_{n\to\infty} \frac{n+2}{n} \cdot (-2)^{\cancel{n}+1-\cancel{n}} \cdot |x-1|^{\cancel{n}+1-\cancel{n}}$$

$1 \cdot (-2)^1 \cdot |x-1|^1 = -2|x-1| = r \quad |-2(x-1)| < 1$

$|2(x-1)| < 1 \rightarrow |x-1| < \frac{1}{2} \rightarrow \boxed{R = \frac{1}{2}}$

21. $\sum_{n=1}^{\infty} \frac{(x+\pi)^n}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{|x+\pi|^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{|x+\pi|^n} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} \cdot |x+\pi|^{\cancel{n}+1-\cancel{n}} = 1 \cdot |x+\pi|^1 = |x+\pi| < 1$$

$\boxed{R=1}$